REQUIREMENTS FOR LOCAL PLANNING TO COVER HAZARDS OF FALLOUT

VOLUME II

APPENDICES TO FINAL REPORT

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VOLUME II

APPENDICES TO FINAL REPORT

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APPENDIX 1

Development of Warning Model

A model to determine the expected effectiveness of a community civil defense warning system is described in this appendix.

I. THEORETICAL DEVELOPMENT OF MODEL

The following definitions are required:

E The event that at least one person in a household or business establishment is listening to a radio or television set with enough attentiveness so that he would hear and understand the meaning of a warning if one were transmitted.

F The event that no one in a household or business establishment is listening to a radio or television set, but there is at least one person in the household or business establishment who has available a radio or television set which could be used to receive messages.

N The event that at least one person in a household or business establishment receives an alert from a NEAR installation.

S The event that at least one person in a household or business establishment is within reception range of a civil defense siren.

W The event that a person receives a warning message or signal, believes it, and acts accordingly.

Events E, N, F, and S are assumed to be independently distributed.

The total population is thought of as being in one of three locations: (a) in households, (b) in business establishments, or (c) in transit. Households include all domiciles. A business establishment includes all places of work and also schools and churches. People in transit are those who are driving, walking, in playbrounds, and all those who for one reason or another are not in a household or business establishment.

It is assumed that if a person receives a believable warning he will spread the word to every person in his household or business establishment. It is assumed that all persons alerted in this manner would also believe in the validity of the warning and act accordingly.

The requirements that must be met in order to have the event W occur may be specified in a number of ways. Two different ways of specifying these conditions are considered and are designated W_1 and W_2.

$\underline{W_1}$

Let W take place if,

(a) A person hears a warning over television <u>or</u> radio, or

(b) A person hears a warning signal from a siren <u>and</u> from a NEAR device, or

(c) A person hears a warning signal from a siren <u>and</u> then turns on his radio or television set and receives a confirmation of the siren warning, or

(d) A person hears a warning signal from a NEAR set <u>and</u> then turns on his radio or television set and receives a confirmation of the NEAR warning signal.

These conditions may be stated mathematically as follows:

$$W_1 = [(E) \cup (SN) \cup (SF) \cup (NF)]$$

It is assumed that if a person hears either a signal from a siren or a NEAR set and if he has a radio or television set available, he will turn on the radio or television set to ascertain the meaning of the siren or NEAR signal.

The probability of being warned under these conditions is given by:[1]

$$p(W_1) = p(E) + (A_1 - A_2 + A_3)(1 - p(E))$$

where:
$A_1 = p(NF) + p(NS) + p(SF)$
$A_2 = p(SN)p(FN) + p(SF)p(SN) + p(SF)p(FN)$
$A_3 = p(SN)p(FN)p(SF)$

[1] This expression is derived in Appendix 1a.

$\underline{W_2}$

Let W take place if,

(a) A person hears a warning over television or radio, or

(b) A person hears a warning signal from a siren, or

(c) A person hears a warning signal from a NEAR set.

These conditions may be stated mathematically as follows:

$$W_2 = (E \cup S \cup N)$$

The probability of being warned under these conditions is given by:

$$p(W_2) = B_1 - B_2 + B_3$$

where:

$$B_1 = p(E) + p(S) + p(N)$$
$$B_2 = p(EN) + p(ES) + p(NS)$$
$$B_3 = p(ENS)$$

II. APPLICATION OF THE MODEL

The p(W) expressed by the model may be interpreted as the expected proportion of the population that will receive a believable warning. The model may be used to assess the effectiveness of civil defense policies to install warning sirens or NEAR units in a community under various assumed conditions required for a warning. The requirements for W_1 may be thought of as assumptions applicable when the public expectation of nuclear war is relatively low. The requirements for W_2, on the other hand, represent an ideal state of awareness which probably can never be reached.

A telephone survey was conducted in the City of Stamford with the consent of the Stamford Office of Civil Defense to determine values of the parameters p(E) and p(F) required in the model. The telephone calls were made between the hours of 9:30 - 11:30 a. m. and 1:30 - 3:30 p. m., on weekdays from the 12th to the 20th of August. The object was to find typical values for the working hours of a summer

day in Stamford. On the basis of 468 answered telephone queries in which the disposition of over 2,000 people was directly or indirectly determined, estimates for p(E) and p(F) were found to have the following values:

$$p(E) = .23$$
$$p(F) = .20$$

These figures may be interpreted as follows. Of the population in Stamford not in transit, 23% are in a household or business establishment where someone is listening to a radio or television set with sufficient attentiveness to hear and believe a disaster warning if one were broadcast. Of the people in Stamford who are in a household or business establishment where no one is listening to a radio or television set, 20% of them have a television or radio in the immediate vicinity that could be turned on.

Using the estimates of p(E) and p(F) established by the survey, the model was used to calculate $p(W_1)$ and $p(W_2)$ for various hypothetical values of p(S) and p(N). The results are presented in Tables 1 and 2.

The effectiveness of the warning system is a strong function of time since the location of people with respect to warning devices varies considerably throughout time. The physical factors which govern the range and effectiveness of individual devices also depend upon the time of day, season of the year, etc.

To illustrate the importance of time as a variable, data was taken from a study conducted in Chicago[1] to estimate p(E) and p(F) for the hours 10:30 p. m. and 2:00 a. m.

The calculations and assumptions used to estimate p(E) and p(F) from the data are given in Appendix 1b. The following values were estimated:

	10:30 p. m.	2:00 a. m.
p(E)	.52	.06
p(F)	.45	.87
proportion of population in transit	.07	.02

Using these values in the model, $p(W_1)$ and $p(W_2)$ were calculated and are presented in Tables 3, 4, 5, and 6.

---

[1] Raymond W. Mack, George W. Backer, The Occasion Instant. The Structure of Social Responses to Unanticipated Air Raid Warnings. Disaster Study No. 5, Publication 945, National Academy of Sciences--National Research Council, Washington 25, D. C., 1961.

Table 1

Probability of Being Warned[1] $p(W_1)$, on a Typical Summer Working Day in Stamford, Connecticut, as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)	0%	20%	40%	60%	80%	100%
100%	.38	.53	.66	.78	.90	1.00
80%	.35	.48	.59	.70	.80	.90
60%	.32	.43	.53	.62	.70	.78
40%	.29	.37	.45	.53	.59	.66
20%	.26	.32	.37	.43	.48	.53
0%	.23	.26	.29	.32	.35	.38

Percent of Population Alerted by a NEAR Device p(N)

Table 2

Probability of Being Warned[1] $p(W_2)$, on a Typical Summer Working Day in Stamford, Connecticut, as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)	0%	20%	40%	60%	80%	100%
100%	1.00	1.00	1.00	1.00	1.00	1.00
80%	.85	.88	.91	.94	.97	1.00
60%	.69	.75	.82	.88	.94	1.00
40%	.53	.63	.72	.82	.91	1.00
20%	48	.51	.63	.75	.88	1.00
0%	23	.48	.53	.69	.85	1.00

Percent of Population Alerted by a NEAR Device p(N)

[1] People in transit not included.

Table 3

Probability of Being Warned[1] $p(W_1)$ at 10:30 p. m. (Chicago) as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)						
100%	.74	.81	.87	.93	.97	1.00
80%	.69	.77	.82	.88	.93	.97
60%	.65	.72	.78	.84	.88	.93
40%	.61	.67	.73	.78	.82	.87
20%	.56	.62	.67	.72	.77	.81
0%	.52	.56	.61	.65	.69	.74
	0%	20%	40%	60%	80%	100%

Percent of Population Alerted by a NEAR Device p(N)

Table 4

Probability of Being Warned[2] $p(W_1)$ at 2:00 a. m. (Chicago) as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)						
100%	.88	.92	.95	.98	.99	1.00
80%	.71	.80	.87	.93	.97	.99
60%	.55	.67	.74	.86	.93	.98
40%	.39	.53	.66	.74	.87	.95
20%	.22	.35	.53	.67	.80	.92
0%	.06	.22	.39	.55	.71	.88
	0%	20%	40%	60%	80%	100%

Percent of Population Alerted by a NEAR Device p(N)

---

[1] The 7% of the population estimated to be in transit is not included.

[2] The 2% of the population estimated to be in transit is not included.

Table 5

Probability of Being Warned[1] $p(W_2)$ at 10:30 p.m. (Chicago) as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)						
100%	1.00	1.00	1.00	1.00	1.00	1.00
80%	.90	.92	.94	.96	.98	1.00
60%	.81	.85	.88	.92	.96	1.00
40%	.71	.77	.83	.88	.94	1.00
20%	.62	.69	.77	.85	.92	1.00
0%	.52	.62	.71	.81	.90	1.00
	0%	20%	40%	60%	80%	100%

Percent of Population Alerted by a NEAR Device p(N)

Table 6

Probability of Being Warned[2] $p(W_2)$ at 2:00 a.m. (Chicago) as a Function of the Percentage of the Population which is Alerted by a Siren and a NEAR Device.

Percent of Population Alerted by Siren p(S)						
100%	1.00	1.00	1.00	1.00	1.00	1.00
80%	.81	.85	.89	.93	.96	1.00
60%	.62	.70	.77	.85	.92	1.00
40%	.44	.55	.66	.77	.88	1.00
20%	.25	.40	.55	.70	.85	1.00
0%	.06	.25	.44	.62	.81	1.00
	0%	20%	40%	60%	80%	100%

Percent of Population Alerted by a NEAR Device p(N)

---

1 The 7% of the population estimated to be in transit is not included.

2 The 2% of the population estimated to be in transit is not included.

Appendix 1a

By definition:

$$W_1 = [(E) \cup (NF) \cup (NS) \cup (SF)]$$

then: $$p(W_1) = p(E) + (A_1 - A_2 + A_3)(1 - p(E))$$

where: $$A_1 = p(NF) + p(NS) + p(SF)$$

$$A_2 = p(NS)p(NF) + p(SF)p(NS) + p(SF)p(NF)$$

$$A_3 = p(SN)p(FN)p(SE)$$

Proof:

$$p(W_1) = p\ [(E) \cup (NF) \cup (NS) \cup (SF)]$$

call $[(NF) \cup (NS) \cup (SF)] = B$

then by substitution:

$$p(W_1) = p(E \cup B)$$

$$= p(E) + p(B) - p(E)p(B)$$

$$= p(E) + p(B)\ [1 - p(E)]$$

It remains to be shown that:

$$p(B) = A_1 - A_2 + A_3$$

By definition:

$$B = [(NF) \cup (NS) \cup (SF)]$$

then: $$p(B) = p\ [(NF) \cup (NS) \cup (SF)]$$

call $[(NS) \cup (SF)] = C$

then: $$p(B) = p\ [(NF) \cup (C)]$$

$$= p(NF) + p(C) - p(NF)p(C)$$

But: $p(C) = p(SN) + p(SF) - p(SN)p(SF)$

Substituting this expression for p(C) in above:

$$p(B) = p(NF) + [p(SN) + p(SF) - p(SN)p(SF)] -$$
$$p(NF)[p(NS) + p(SF) - p(NS)p(SF)]$$

Rearranging terms:

$$p(B) = p(NF) + p(NS) + p(SF) - p(NS)p(NF) -$$
$$p(SF)p(NS) - p(SF)p(NF) + p(SN)p(FN)p(SE)$$
$$= A_1 - A_2 + A_3$$

Appendix 1b

The following values were taken from The Occasion Instant, pages 29 and 30:

What people were doing at 10:30 p.m.

42%	Watching television
13%	Conversing
13%	Preparing for bed
8%	Listening to the radio or reading

Remaining: Household chores, Driving, Walking, etc. — All of the activities less than seven percent ea

100%

From the above values, the following more detailed breakdown was constructed:

42%	Watching television
13%	Conversing
13%	Preparing for bed
4%	Reading
4%	Listening to the radio
5%	Household chores, sleeping, miscellaneous
81%	
7%	Driving and walking
12%	Working
100%	

p(E) was calculated as follows:

$$\begin{aligned} p(E) &= .42 && \text{Proportion watching television} \\ &+ .04 && \text{Proportion listening to radio} \\ &+ .06 && \text{Half of the working population} \\ &= .52 && \text{at 10:30 p.m.} \end{aligned}$$

p(F) was calculated as follows:

From 1960 census data:

.915 proportion of households have a radio
.863 proportion of households have a television

Assuming independent distribution of radios and televisions, the probability that a household will have a radio or a television set is given by:

$$p(R \cup T) = .915 + .863 - (.915)(.863)$$

$$= .98$$

each This means approximately 98% of the households do have either a radio or television set.

From above, 42% plus 4% are listening to radio or television. This leaves 54% who are not listening to radio or television. Out of this 54%, 7% are in transit, leaving 47%. p(F) is now arbitrarily taken to be a high percentage of this remainder.

$$p(F) = .45 \quad \text{at 10:30 p.m.}$$

Parameters at 2:00 a.m.

Estimate of what people are doing from Chicago data:

12%	Working
2%	Driving or walking (in transit)
86%	At home

p(E) = No listeners at home and half of the workers

= 0 + .06

p(E) = .06

p(F) = 98% at home have radio or television. Say half of working population not listening have radio or television available

$$p(F) = (.98)(.86) + 1/2\ .06$$

$$p(F) = .87$$

APPENDIX 2

Development of Assignment Model

APPENDIX 2

I. INTRODUCTION

In any community it is likely that there will be some areas where there are more people than shelters, and other areas where there are more shelters than people. It is also likely that, unless something is done to prevent it, excessive and unnecessary casualties will result from overcrowded conditions in some shelters while others remain vacant and from the exposure of people to lethal doses of radiation while searching aimlessly for available shelter.

In this appendix a technique is presented for assigning persons from areas of excess population to areas of excess shelter in such a way as to minimize the over-all distance traveled and to equalize the degree of overloading, if any, on the shelters in the community. This is done on the assumption that distance is proportional to time. (It is desired to minimize travel time but not practical to do so with available data at this time.) It is also assumed that under actual emergency conditions people will not be turned away from any shelter which can possibly accommodate them.

Distance was selected as a criterion rather than time to shelter primarily because of the greater confidence which could be placed in its measurement. The existing network of roads and topography of the area make a limited number of routes from origin to destination feasible, and the distance which must be traveled can therefore be estimated with precision and confidence. The time which would be required, on the other hand, would be affected by whether people walked, drove, or were taken to shelter, and by the type and extent of traffic control and direction--factors which are highly variable and subject to change by decree. It was felt best, therefore, to assign people on the basis of distance, and determine the times required to traverse that distance as a function of transportation policies in a later analysis. The means of determining the time required to reach shelter under different transportation conditions is discussed in Appendix 3.

II. PROCEDURE

The procedure includes: (1) the collection of data on the population of the community, the number of shelters in it, and the various routes which might be used by people seeking shelter; (2) the calculation of an overload

factor where the total population exceeds the total number of shelters, the adjustment of the number of shelters in each tract using this overload factor, and the determination of the number of excess people or excess shelters for each area; and (3) the assignment of persons from areas of excess population to areas of excess shelter capacity, using rules which assure at least a "good" solution to the problem. These three steps are discussed more fully under separate headings below.

A. Data Sources

The principal sources of data used in this analysis were the 1960 Census of the United States (6), the final report of Phase II of the National Fallout Shelter Survey (2), and a map of the City of Stamford, Connecticut. Because the Census Bureau data were reported by census tracts and the shelter survey data were reported by National Location Code locations, it was necessary to get the correspondence between the two. This information is available for all regions in a series of volumes prepared for OCD and OEP by the Bureau of the Census (5). The National Fallout Shelter Survey Report Formats and Descriptions of Content (3) was found to be indispensable in interpreting the data in the report itself.

Population. Assignments were based on resident population but could equally well have been based on daytime or nighttime population. Resident populations are given in references 2, 5, and 6, but those in reference 6 are suspect, a machine error having been found in some of the readouts (4). Daytime and nighttime populations are given only in the survey report (2) and then only for those locations which have some potential shelter spaces. These can be estimated, where necessary, from other data contained in the census reports (6).

Shelters. In the Stamford assignments, total possible shelters with a protection factor of 100 or greater (after improvements) were used. The data were taken from reference 2, where they were found in "Summary A" for each individual tract under the heading "TOT POSS SPACES." (See reference 3, page 5, row 24 under column 23.) Depending on the purpose to be served, data for total existing spaced with a protection factor of 100 or better or for total existing spaces with a protection factor of 40 or better could have been used.

Routes to Shelter. Maps of the city were obtained from the city engineering department. These were readily available for all cities contacted and provided sufficient detail for the analyst to draw in the tract boundaries, pick out by eye a reasonable route from each origin to each

destination, and measure the distance along each such route. There will be times, of course, when unique local conditions which are not clear on the map will make a choice a bad one, and the services of a local person could be used to advantage. This situation was not encountered in the selection of shelter-seeking routes for Stamford, Connecticut.

B. Data Processing

Overload Factor. In many communities there will be fewer total shelters than persons. For these communities an overload factor is computed. It is the ratio of the total population to the total number of shelters, or, the average number of persons per shelter. This factor is applied to the number of shelters in each tract to get a new, "adjusted," number of shelter spaces. For communities with at least as many shelter spaces as people, the adjustment factor is 1.0. All assignments are based on these adjusted numbers of shelter spaces.

Shelter/Population Excesses. For each tract the difference between the adjusted number of shelter spaces and the tract population is taken to get the number of excess persons or the number of excess shelters for that tract. Tracts with excess population then become origins, and tracts with excess shelters become destinations.

III. SETTING UP THE PROBLEM

Table 1 presents the basic data for the problem in matrix form. There are sixteen areas of excess population and eight areas of excess shelters, for a total of twenty-four areas. In this particular case, the number of excess shelters, 44,397, is exactly equal to the number of excess persons. Were this not so, a hypothetical seventeenth origin would be added to the matrix to make the number of persons equal to the number of shelters.

The excess populations and shelter spaces, where they exist, are shown in the marginal row and column of Table 1. The distance in arbitrary units from each area of excess population to each area of excess shelter capacity is given in the body of the table. The problem is to assign the excess population to the excess shelters in such a way as to minimize the total number of man-units traveled in reaching shelter. This is the standard "Transportation Problem" which is of the form:[1]

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[1] See Gass (reference 1, pages 6-8) or any standard text on linear programming for a complete statement of this problem.

Table 1

Stamford, Connecticut
Distance From Areas of Excess Population to Areas of Excess Shelter (Arbitrary Units)[1]

Origin	Destination 1	11	12	16	17	18	22	23	Excess Population
2	143	140	122	136	150	146	163	163	1,396
3	157	139	120	150	164	160	177	183	2,646
4	104	85	73	97	111	107	124	134	2,464
5	83	63	31	56	70	66	83	67	2,864
6	64	44	31	57	71	67	94	87	3,768
7	118	61	58	107	117	103	132	121	2,355
8	36	28	13	52	62	48	77	85	2,559
9	73	32	39	62	72	58	87	104	1,958
10	58	21	26	47	57	43	72	86	2,180
13	22	29	12	15	29	25	42	42	2,987
14	21	55	47	32	43	36	23	10	1,234
15	6	39	41	20	31	21	11	9	4,351
19	35	41	60	50	31	22	37	53	4,535
20	41	36	56	47	26	17	37	54	2,813
21	16	38	44	34	9	18	17	31	3,824
24	40	71	78	68	43	52	42	57	2,463
Excess Shelters	29,091	318	3,392	1,914	2,431	673	1,933	4,645	44,397

[1] Readings proportional to distance, derived from a mechanical device used to measure miles on a map.

a_i = excess population in area i

b_j = excess shelters in area j

C_{ij} = distance from area i to area j

X_{ij} = number of excess persons from area i assigned to area j

Where it is desired to select X_{ij} such that $\sum_{ij} X_{ij} C_{ij}$ is a minimum subject to the constraints:

$$\sum_i a_i = \sum_j b_j$$

$$\sum_i X_{ij} = b_j$$

$$\sum_j X_{ij} = a_i$$

IV. SOLUTION OF THE PROBLEM

Two totally different methods of solving the problem were found feasible, with their desirability depending on the time available and the degree of accuracy required. For preliminary checks and estimating purposes, a heuristic "eyeball" technique provided a solution which, in this particular case, was only 5% or so less than optimum, and which took less than one-tenth the time required to reach an optimum solution. Such good results, however, are not to be expected every time, and one must not lose sight of the fact that this technique provides no objective means of determining just how good a solution is. Therefore, for more precise work, as in planning and making actual assignments, the time required to reach an optimum by more formal means is not only well spent, but the only sound policy. For the optimum solution, a technique described by Gass (reference 1, pages 145-152) was employed. Although it took longer, it was simple enough that clerical help could use it without difficulty to solve the problem, and it had the advantage common to formal mathematical techniques that it measured its own accuracy. Each of these techniques is discussed briefly below.

Heuristic Method. To get an "eyeball" solution, one scans the rows and columns of the matrix looking for a cell of least cost. When such a cell is found, as many excess persons as possible are assigned to that shelter area. When no cells of least cost remain to which assignments can be made, the matrix is scanned to determine the least costly place to assign the remaining excess persons. When all of these excess persons have been assigned, the matrix is scanned to determine whether any adjustments can be made in the assignments to reduce the over-all cost. No attempt is made to make all such adjustments, because to do so would be to seek an optimum solution which can much more easily and reliably be done using a formal approach; instead, only the "obvious" changes are made, and a "good" rather than the "best" solution is accepted.

Optimum Method. Using this method, one starts with what is called a "basic feasible solution." This is a solution which assigns all of the excess people to all of the excess shelters in exactly $m + n - 1$ assignments, where m is the number of areas of excess shelters and n is the number of areas of excess population. This need not be a good solution and can be obtained by a very simple technique called the "Northwest Corner Rule" (reference 1, page 146). Starting with this solution, each iteration requires first mn algebraic sums of the form $a + b - c$ to determine which cells should be adjusted; then one or two adjustments involving the removal of persons previously assigned to one area and their reassignment elsewhere subject to the original constraints. For a city the size of Stamford, approximately twenty iterations would be required, and the entire manual process would take on the order of two and one-half working days. Computer programs are available for this problem, and commercial computer organizations exist which will deliver a solution for two to three hundred dollars per matrix.

The Solution. Table 2 presents the solution to the Stamford problem obtained using the optimum technique: no eyeball solution is presented. This solution is not necessarily unique, but it is the best obtainable. The over-all number of man-miles traveled by those seeking shelter outside of their own area is 101, 109, and the average individual distance traveled is 2.28 miles. This means that at 3 miles per hour walking speed 50% of the excess persons will be sheltered within three-quarters of an hour. Assuming that all of those who are assigned to their own areas find shelter within this time, with no other form of transportation involved, over 75% of the total population of Stamford will be sheltered within 45 minutes.

Table 2

Stamford, Connecticut
Assignment of Persons From Areas of Excess Population
to Areas of Excess Shelter

Origin	Destination							
	1	11	12	16	17	18	22	23
2	1396							
3	2646							
4	2464							
5	2864							
6	3768							
7		318	2037					
8	1204		1355					
9	1958							
10	266			1914				
13	2987							
14								1234
15	940							3411
19	4535							
20					2140	673		
21	1600				291		1933	
24	2463							

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APPENDIX 3

Assignment of Shelters by Census Tract

I. INTRODUCTION

Within a given civil defense area persons can be rationally assigned from census tracts of excess population to census tracts of excess shelter capacity using the techniques of linear programming (2). Given these assignments, it is possible to determine the pattern of traffic for the community under certain assumptions and to express this pattern in terms of number of persons in the shelter area versus time. This appendix presents a technique for accomplishing this latter task, using Stamford, Connecticut, as a case example.

The assumptions used in this analysis are:

A_0: All persons will travel by private automobile or walk;

A_1: All persons moving entirely within a census tract will travel on foot;

A_2: Persons moving from one tract t› another will travel by automobile if:

a. The straight-line distance from the center of the origin tract to the center of the destination tract is greater than one mile, and

b. there is at least one car in the household unit.[1]

A_3: No more than one car from each household unit will be used;

A_4: Should the number of persons assigned to tracts beyond one mile exceed the number of persons in household units having one or more cars, such excess persons are assumed to ride with persons having automobiles;

A_5: Traffic moves from origin to destination exactly as assigned by the linear program;

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[1] A "household unit" generally corresponds to a family unit. See reference 3 for a precise definition of this term.

A_6: All traffic from a given origin will travel in a wave of constant length determined by the constant speed of the automobile, the average length of cars in the wave, the number of cars in the wave, and the constant separation between cars in the wave;

A_7: At each intersection, the first wave to arrive will occupy the intersection until it has completely passed. Other waves waiting at the intersection may enter only when the wave has passed or when a gap appears in it. When a waiting wave so enters an intersection, it will continue to occupy it until it is completely through or until another waiting wave enters at a gap.

A_8: When part of a wave must be diverted to a proximate origin, that part shall be the last part of the wave.

A_9: The effects of starting and stopping times and road obstructions other than moving cars will be neglected.

II. SETTING UP THE PROBLEM: BASIC DATA

Three basic items are required to solve this problem: (1) a set of assignments of excess persons who will walk and of excess persons who will drive from origin tracts to destination tracts; (2) information concerning the availability of automobiles in the origin tracts; and (3) information about the transportation routes, including the distances between points on the automobile routes, and the maximum and minimum distances walked by those on foot.

The assignment of excess persons to tracts having excess shelter capacity is accomplished by means of the computational procedure discussed in Appendix 2. The determination of who will walk and who will ride is accomplished by measuring the straight-line distance from center of origin tract to center of destination tract. Where the distance is one mile or greater, automobiles are used; where the distance is less than one mile, the excess population moves on foot.

The number of cars departing from each origin is determined by taking from Reference 3 the total number of housing units having one or more automobiles available and comparing it with the number of housing units whose occupants will leave the origin. The smaller of these numbers represents the number of cars leaving the origin. That is, for each origin:

$$A_r = \text{Min}\left[A_a; \frac{H_o P_e}{P_r}\right]$$

Where: A_r is the number of cars on the road;

A_a is the number of occupied housing units with automobiles;

H_o is the number of occupied housing units;

P_e is the excess population;

P_r is the resident population.

The number of cars going to each of several destinations from a given origin is assumed proportional to the number of people assigned to each destination from that origin. That is:

$$A_{ij} = \frac{A_i P_{ij}}{P_i}$$

Where: A_{ij} is the number of automobiles traveling from origin i to destination j

A_i is the total number of cars leaving origin i

P_{ij} is the number of excess people from origin i assigned to destination j

P_i is the excess population of origin i

As soon as the assignments of excess persons to shelter areas is known, transportation routes can be determined and distances measured. Walking routes are measured from a point on the periphery of the destination tract along the shortest possible path to the nearest and then to the most distant point in the origin tract, giving two figures which can later be converted, at any assumed walking rate, to times of first and last arrival of foot traffic from the given origin.

Automobile routes must be more carefully thought out. Traffic is generally assumed to flow from the small roads and lanes onto the larger routes and main arteries; but exceptions are made where the number of intersections can be reduced, where a parallel flow of traffic can be established, or in general where the rapid flow of traffic will be aided.

It is necessary to measure the distance between each two adjacent points along the routes, where a point is a destination, an origin, or an intersection of two routes. The over-all distances measured are between the center of the origin tracts and the center of the corresponding destination tracts.

III. SOLUTION OF THE PROBLEM

The problem is first solved for automobile traffic alone, then for foot traffic. These are then combined to provide the over-all arrival rates and times for each destination tract.

In presenting the technique used in solving the problem, Stamford, Connecticut, is used as an example. The Assignment Matrix for Stamford appears in Table 1. The necessary automobile data are shown in Table 2. The over-all flow of traffic is presented in schematic form in Figure 1, All calculations are for the single case of 20 mph driving speed, 3 mph walking speed, and single lane vehicular traffic.

A. Automobile Traffic

The first step is to determine the "wavelength" of the cars leaving each origin. This is the interval of time, in hours, the moving wave of cars requires to pass a stationary point. It is given by the expression:

$$W_{sij} = \frac{N_{ij}L}{5280s}\left(1 + \frac{ns}{10}\right)$$

Where: W_{sij} is the wavelength, in hours, at speed s, of the wave travelling from origin i to destination j.

N_{ij} is the number of cars in that wave.

L is the unit length of the cars, in feet.

s is the speed at which the cars are travelling, in mph.

n is the number of car-lengths separation between cars in the wave, in feet per 10 mph of speed.

For L = 16, S = 20, and n = 1.5, this is numberically equal to $(6.06 \times 10^{-4})(N_{ij})$, and Table 3 is obtained by a series of constant multiplications.

Table 1

Stamford, Connecticut
Assignment of Persons from Areas of
Excess Population to Areas of Excess Shelter

Origin Tract \ Destination Tract	1	11	12	16	17	18	22	23
2	1396							
3	2646							
4	2464							
5	2864							
6	3768							
7		318	2037					
8	1204		(1355)					
9	1958							
10	266			1914				
13	(2987)*							
14								1234
15	(940)							(3411)
19	4535							
20					2140	(673)		
21	(1600)				(291)		(1933)	
24	2463							

*Numbers in parentheses indicate persons who will walk because of proximity of origin and destination tracts.

Table 2

Stamford, Connecticut
Automobile Data

Origin Tract	Automobile Available (A_a)	Housing Units Occupied (H_o)	Resident Population (P_r)	Excess Population* (P_e)	$\frac{H_o P_e}{P_r}$	A_r**	$\frac{P_e}{A_r}$***
2	378	378	1396	1396	378	378	3.69
3	691	740	2646	2646	740	691	3.83
4	660	660	2464	2464	660	660	3.73
5	782	787	3056	2864	737	737	3.89
6	1075	1085	4007	3768	1020	1020	3.69
7	845	860	3326	2355	609	609	3.87
8	799	799	3225	1204	298	298	4.04
9	843	898	3211	1958	547	547	3.58
10	691	756	2646	2180	623	623	3.50
14	1322	1503	6015	1234	308	308	4.01
19	1147	1223	4535	4535	1223	1147	3.95
20	859	903	3030	2140	638	638	3.35
24	612	658	2463	2463	658	612	4.02

*Includes only those persons assigned to shelter areas more than one mile distant.
**cars on road
***average persons per car

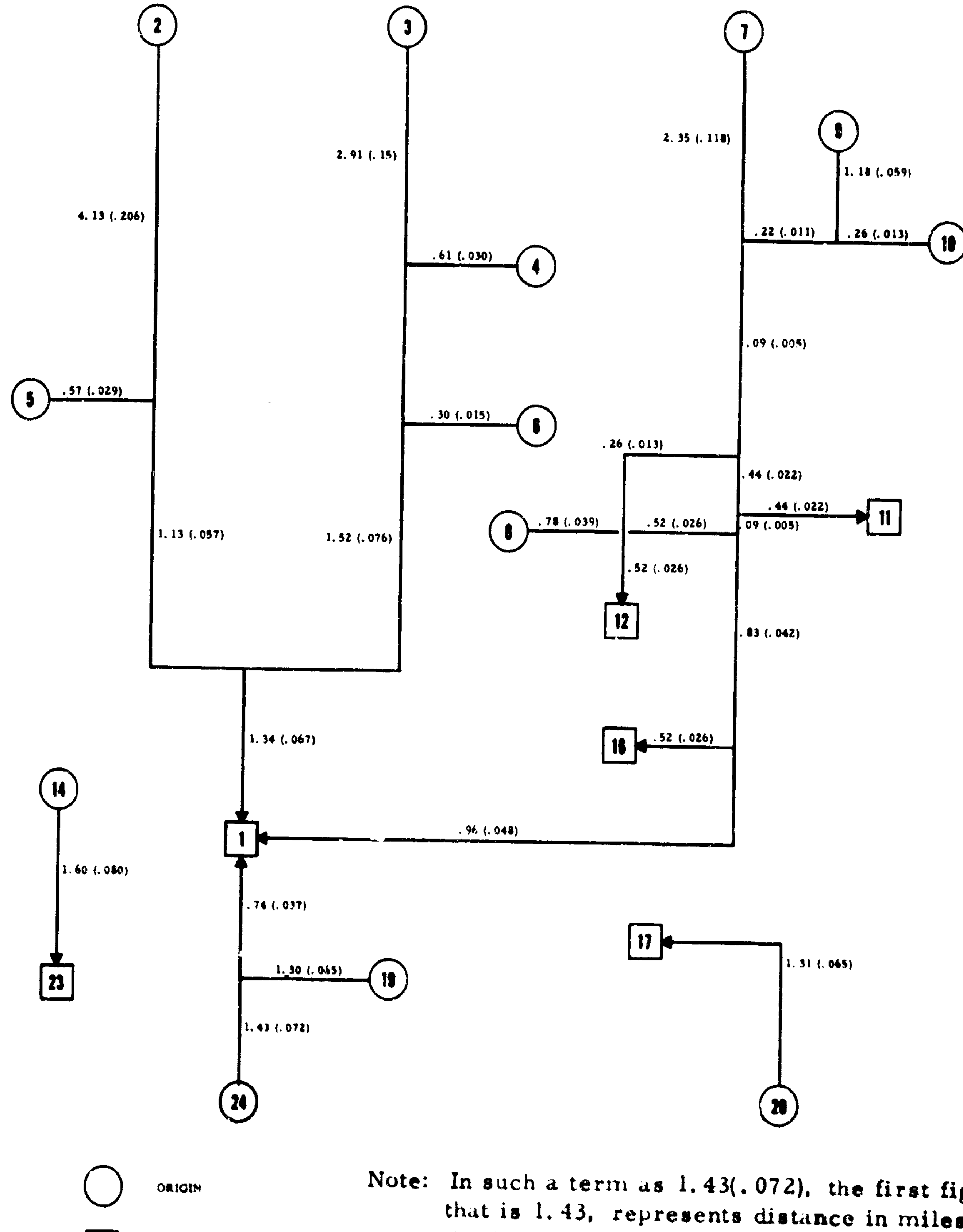
Table 3

Stamford, Connecticut
Wavelengths by Origin and Destination

s = 20 mph; L = 16 ft.; n = 1.5

Origin (i)	Dest. (j)	No. of Cars (C_{ij})	Wavelength (W_{20ij})	Rate (persons per hour)
7	11	82	.050	6,385.50
7	12	527	.319	6,385.50
8	1	298	.181	6,666.01
9	1	547	.331	5,907.00
10	1	75	.046	5,857.50
10	16	548	.332	5,857.50
14	23	308	.187	6,616.50
19	1	1147	.695	6,517.50
20	17	638	.387	5,527.50
24	1	612	.371	6,663.00

[1]Rate $= \dfrac{C_{ij}}{W_{ij}} \cdot \dfrac{P_e}{A_r}$



Note: In such a term as 1.43(.072), the first figure, that is 1.43, represents distance in miles, the figure is parenthesis, that is (.072) represents time in hours.

Figure 1. Stamford, Connecticut. Schematic Diagram of Automobile Traffic Flow. (Not to scale.)

The second step is to convert all distances to times. This has already been done in Figure 1, where the times appear in parentheses following the distances.

When these two preliminary steps have been completed, the traffic flow can be analyzed by inspection, using a worksheet to indicate the time of arrival of the first and last cars from each origin at each destination.

The flow from origin 14 to destination 23, for example, is simple. The first car will arrive at 23 at .080 hours, and the last will arrive .187 hours later, or at .267 hours.

The flow from origins 2, 3, 4, 5, and 6 is considerable, but is very simple to compute. The first car will arrive from the nearest origin -- 5 -- at .153 hours. Inspection of Figure 1 will show that the flow from each of these five origins will encounter one or more delays on the way to the destination and that, therefore, there will be no gaps in the composite wave entering destination 1. Thus, the arrival time of the first car in the composite wave will be .153 hours, the interval between first cars from consecutive waves will be equal to the lengths of the waves, and the time of arrival of the last car from the last wave will be .153 hours plus the sum of the five wavelengths.

The flow from origins 7, 8, 9, and 10, however, is both considerable and harder to compute. Two complicating factors not previously encountered are present in that pattern: first, the diversion of portions of the composite wave to proximate destinations; and second, the existence of an intersection where traffic crosses without merging. The diversions leave gaps in the wave which might be reduced or eliminated as the wave encounters further delays, or which might be carried intact all the way to the ultimate destination. Where these exist, they present the opportunity for waves already waiting at an intersection to preempt the main artery and alter the sequence of arrivals. The non-merging intersection, on the other hand, must be examined to determine whether the two waves using it will attempt to do so at the same time. If so, an existing gap in one or both waves could be enlarged or compressed; or, at the very least, one of the waves would be delayed in reaching its destination.

Because this pattern illustrates many of the situations encountered in this type of analysis, it will be discussed in some detail.

Inspection of Figure 1 shows that the first car to reach destination 1 will be from origin 10. It will arrive at .146 (hours) and the 75th will arrive at .192. The remaining 548 cars are diverted to 16 where the first arrives at .170 and the last at .502.

Further inspection reveals that waves 9 and 7 will follow, in that order, right on the tail of wave 10, with the first 82 cars from 7 going to 11, and the last 527 going to 12. The first car from 9 will arrive at 1 at .146 + .378 (the entire wavelength of 10) = .524 and the last at .855 because the gap left in wave 10 by the diversion of part of it was unchanged by further delays to wave 10.

The last car from 9 passed the road to 11 at .013 + .011 + .005 + .022 + .378 + .331 = .760 and, therefore, the first would reach 11 at .760 + .022 = .782 hours, and the last at .782 + .050 = .832 hours. The 83rd car from 7 passed the road to 12 at .013 + .011 + .005 + .378 + .331 + .050 = .788 hours, and would arrive at the non-merging intersection .013 hours later, or at .801 hours; and assuming no delay, would arrive at 12 at .026 hours later at .827 hours. The last car from 7 would then arrive at 12 at 1.148 hours.

Now, to check the above assumption: the last car from 9 would pass the point where cars from 8 waited at .765. Therefore, the cars from 8, which had been waiting since .065 to enter the main road, would enter at .765 and would clear the non-merging intersection at .765 + .181 - .026 = .920 hours. From this it follows that the cars from 7 would have to wait at the non-merging intersection .117 hours, so that arrivals at 12 would be at .946 and 1.265 instead of .829 and 1.148.

Since the wave from 8 would enter the main stream right on the tail of the wave from 9, the arrival times at 1 would be .855 and 1.036.

Thus, all arrival times in this pattern would be:

Table 4

Stamford, Connecticut
Summary of Automobile Arrival Time

s = 20 mph, one lane traffic

Origin	Destination	First Car	Last Car
7	11	.782	.832
7	12	.946	1.265
10	16	.170	.502
10	1	.146	.192
9	1	.524	.855
8	1	.855	1.036

B. Foot Traffic

Calculation of walking times and rates is a much simpler proposition. Required data are the area assignments, and the maximum and minimum distances for each origin-destination combination. The distances divided by the walking rate (here taken to be 3 mph) gives the time of arrival of the first and last persons from each origin. Their difference is the arrival interval which, when divided by the number of persons assigned, gives the arrival rate. Table 5 summarizes these data for Stamford, Connecticut. Independence between automobile and pedestrian traffic is assumed.

Table 5

Stamford, Connecticut
Inter-tract Walking Arrivals

			Distance		Time*		(persons/hr.)
Dest.	Origin	Number	Min.	Max.	Start	Interval	Rate
1	13	2987	0	1.13	0	38	7926.90
1	15	940	0	.57	0	19	4989.24
1	21	1600	0	1.17	0	39	4088.80
12	8	1355	0	1.13	0	38	3595.90
17	21	291	0	.52	0	17	1673.25
18	20	673	0	.78	0	26	2579.81
22	21	1933	0	1.19	0	36	5335.08
23	15	3411	0	.82	0	27	12387.39

*At 3 mph, in 1/100 hr. units.

C. Combining the Data

Perhaps the easiest method for bringing all of the arrival times and rates data together for a given tract is to draw a "bar chart" using a separate solid line to indicate on a time scale the interval during which traffic is arriving from each origin. The beginning and end of each such line is a point in time at which the arrival rate will almost certainly change. These points can very easily be picked off such a chart, and the arrival rates for all of the incoming waves summed to give a total arrival rate for each time interval. The end result would then appear as in Table 6, which summarizes the results of all of the calculations for Census Tract 1 in Stamford, Connecticut.

Table 6

Stamford, Connecticut
Summary of All Arrivals in Tract 1

20 mph automobile speed, 1 lane, 3 mph walking speed

Destination	Period (hundredths of hrs.)	Origins	Arrivals (persons)	Cumulative Arrival (persons)	Arrival Rate (persons/ hr.)
1	0 - 10	13, 15, 21	1691	1691	17,005
1	10 - 15	13, 15, 21, 19	1174	2865	23,522
1	15 - 19	13, 15, 21, 19, 10, 5	1412	4277	35,798
1	19 - 20	13, 21, 19, 10, 5	304	4581	30,309
1	20 - 38	13, 21, 19, 5	4509	9090	24,952
1	38 - 39	21, 19, 5	172	9262	17,025
1	39 - 53	19, 5	1831	11093	12,936
1	53 - 59	19, 5, 9	1141	12234	18,843
1	59 - 79	19, 9, 2	3715	15949	18,430
1	79 - 82	9, 2, 24	560	16509	18,546
1	82 - 86	9, 24, 6	746	17255	18,629
1	86 - 104	24, 8, 6	3496	20751	19,418
1	104 - 116	24, 6	1528	22279	12,722
1	116 - 144	6	1702	23981	6,089
1	144 - 184	4	2464	26445	6,154
1	184 - 226	3	2646	29091	6,319

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APPENDIX 4

Glossary of Symbols

Pedestrian Traffic Model

A	area
a	parametric constant
A_c	area of entire community
A_s	area of tracts having excess shelter
k	parametric constant
θ	variable of integration
P	population
P_e	excess population
r	distance from center of core shelter area
r_1	radius of core shelter area
r_2	radius of entire community
t	time
V	walking velocity

Vehicular Traffic Model

b	parametric constant
d_{min}	least distance from an origin to a destination area
H	number of households in origin of greatest excess population having one or more cars available
n	number of roads leading into shelter area
P_e	excess population
P_m	excess population of origin with greatest excess population
P_{mt}	total population of origin with greatest excess population
Q	number of persons per vehicle
R	unit vehicle arrival rate
S	vehicular road speed
t	time
t_i	time of arrival of last vehicle on artery i
t_m	time of arrival of last vehicle
t_o	time of arrival of first vehicle

APPENDIX 4

1. SUMMARY

Two independent analytic models of inter-tract shelter seeking traffic are presented and discussed: one for pedestrian and one for vehicular traffic. Methods for evaluating the model parameters are discussed. Stamford and Waterbury, Connecticut, are used as examples in the derivation and application of the models. The results are compared with the results of the earlier point-by-point simulation analysis of these two cities.

2. INTRODUCTION AND DISCUSSION

2.1 Introduction

The analysis of the flow of pedestrian and vehicular traffic enroute to shelter areas was a basic part of the over-all analysis of shelter-seeking operations discussed in Chapter II of this report. The method used is described in detail in Appendix 3. It is essentially a paper-and-pencil simulation technique, which uses actual locations, distances, and times for each group of people and shelters; traces the flow of traffic along actual routes on a point-by-point basis; and resolves conflicts among groups by means of a clearly defined set of rules which are assumed to reflect realistic emergency operating conditions.

While simple in concept and techniques of execution, simulation is a lengthy and tedious procedure better suited for automatic data processing than for hand computation. Also in the simple form used here, it does little to reduce the problem to its basic elements and provide any real understanding of the relationships between these and what is happening in the real world. For these reasons, work on the two models presented in this appendix was begun. In their current state of development, these models already point to some very basic relationships between community parameters and the flow of traffic to shelter areas. If fully developed and validated, they could provide additional insights into the fundamental processes involved in shelter-seeking operations, and could be used to estimate, with a minimum of calculation and data collection, the flow of traffic for any community.

These two models are introduced in this section of this appendix and the results obtained using them are compared with the results obtained using the previous method. The pedestrian traffic model is then formally derived in Section 3 and the vehicular traffic model in Section 4. Section 5 contains sample calculations for Stamford and Waterbury, Connecticut.

2.2 Results

Figures 1 through 8 on the following pages show the results of the work discussed in this appendix. Figures 1 through 4 show the cumulative arrival curves as determined by both the analytic method developed herein and the point-by-point simulation technique previously developed. Figures 5 through 8 show population density and rate of arrival curves and the data upon which they are based, so that the validity of the basic assumptions regarding population density and distribution may be assessed.

2.2.1 Figures 1 and 2: Cumulative Walking Arrivals

The analytic expression developed to relate walking arrivals to time is:

$$W(t) = P_e \left(\frac{(r_1 + Vt)^{2-a} - r^{2-a}}{r_2^{2-a} - r^{2-a}} \right)$$

for all values of a not equal to 2. In the special cases where a equals 2, the function takes the form:

$$W(t) = P_e \frac{\ln(1+Vt/r_1)}{\ln(r_2 / r_1)}$$

The parameters of this expression are: the number of persons who must seek shelter outside of their own area (P_e); the speed at which these people walk (V); a measure of the area of those tracts having excess shelter (r_1); a measure of the area of the entire community (r_2); and a parametric constant (W).[1]

[1] Further work is required to determine whether this parameter will assume different values for different types of communities. The same value (a=2) was used herein for Stamford and Waterbury with equally good results

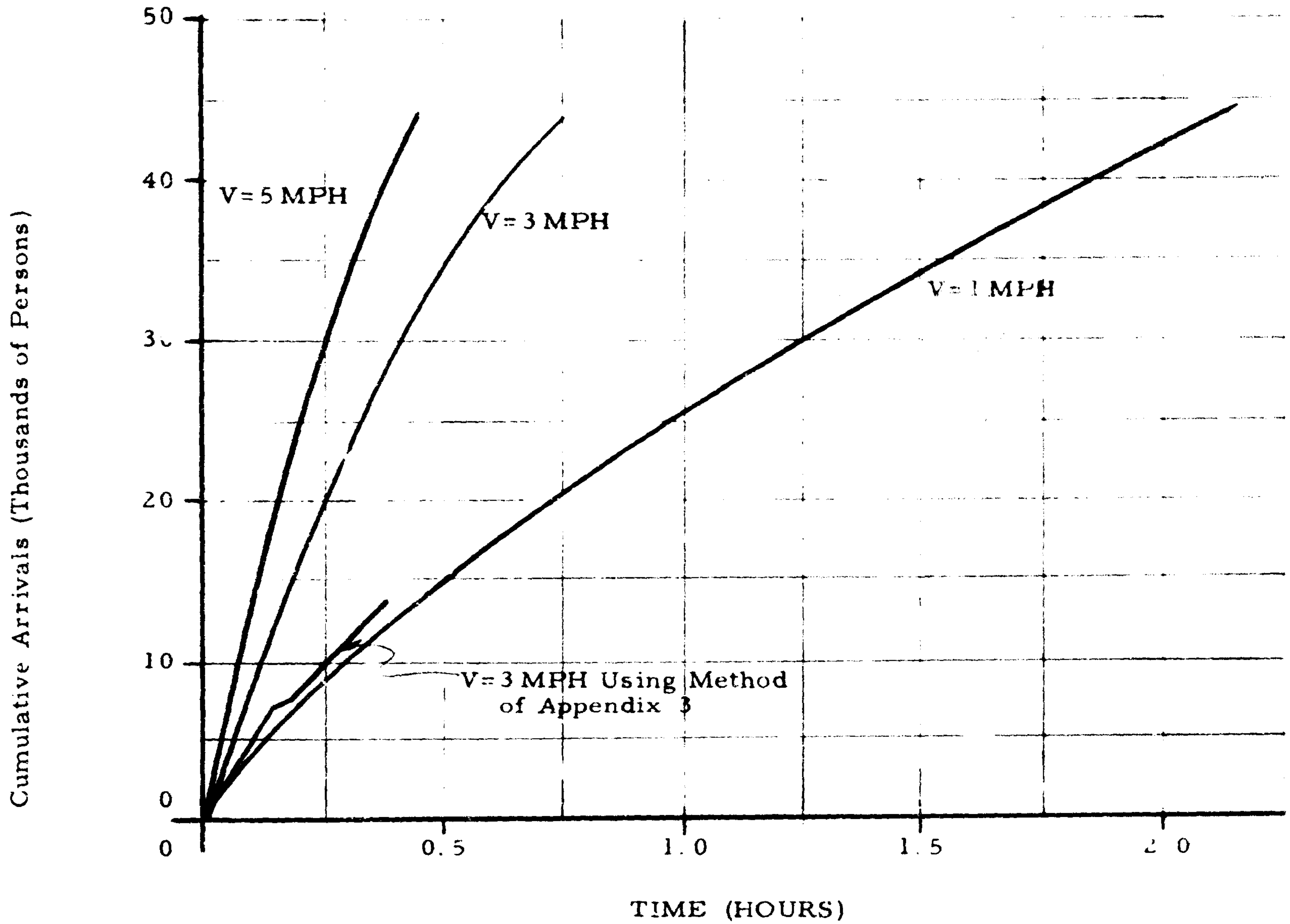

Figure 1. Cumulative Pedestrian Arrivals in Shelter Area vs. Time--Stamford, Connecticut

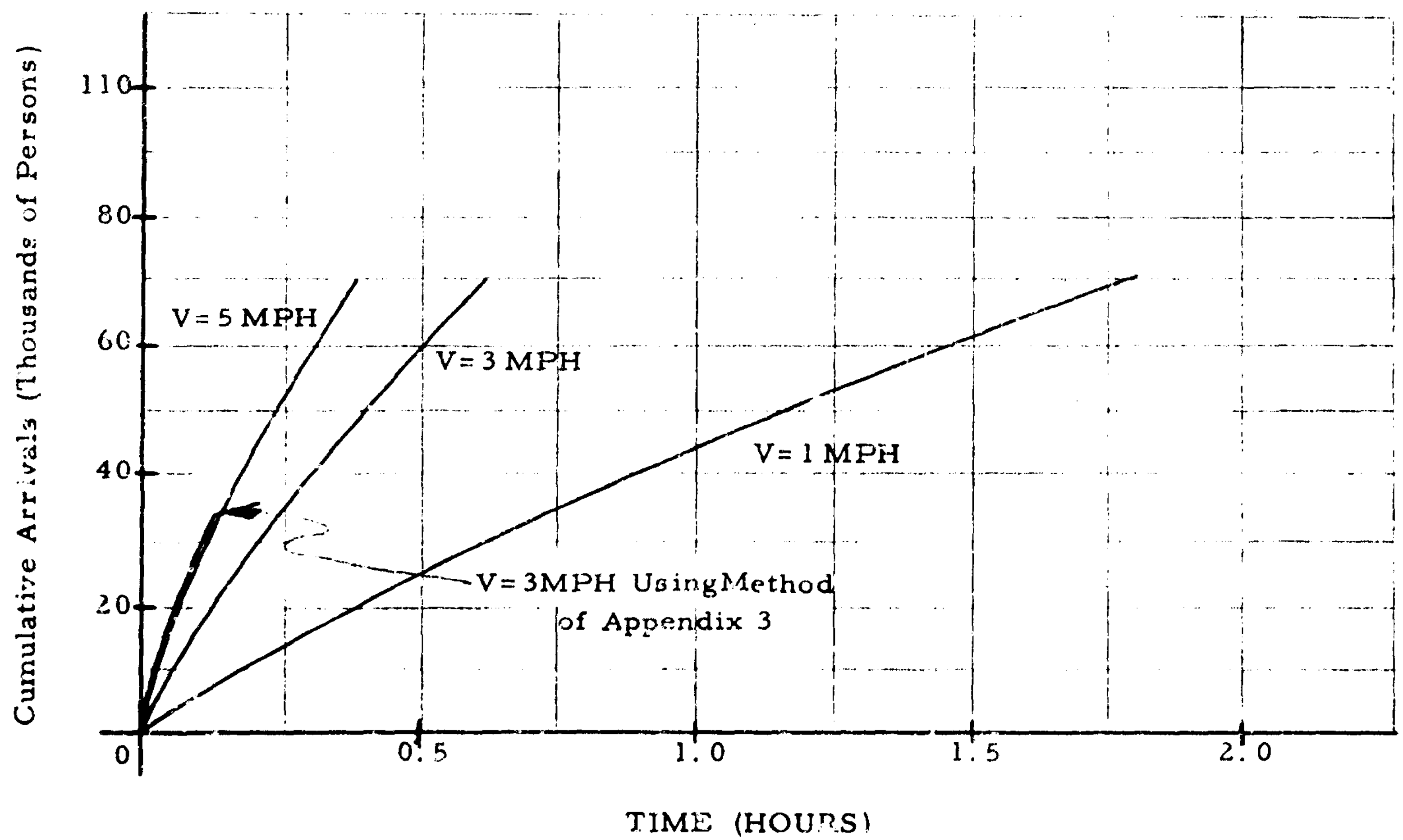

Figure 2. Cumulative Pedestrian Arrivals in Shelter Area vs. Time--Waterbury, Connecticut

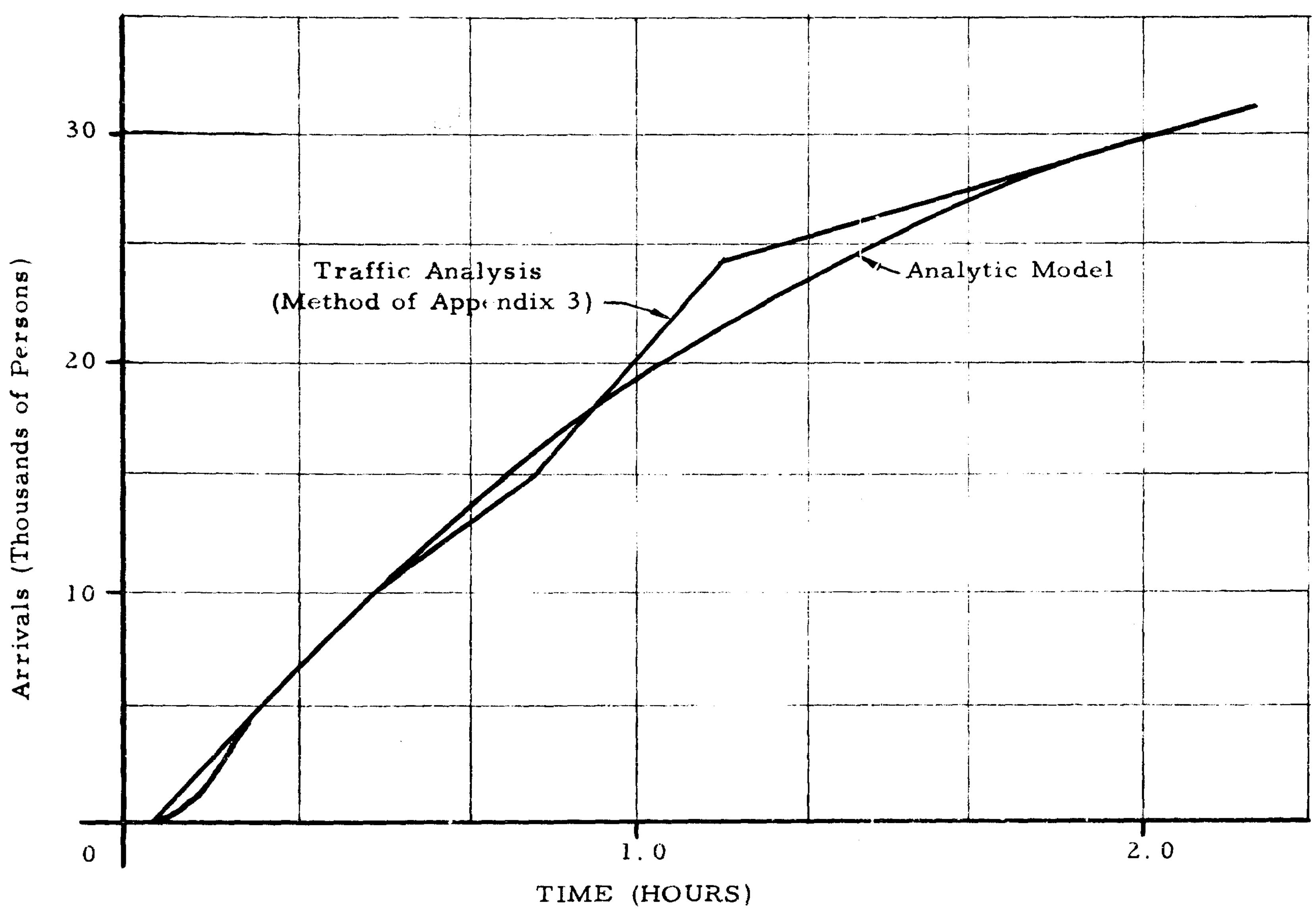

Figure 3. Stamford, Connecticut, Cumulative Vehicular Arrivals in Shelter Areas vs. Time: 20 MPH One Lane Traffic.

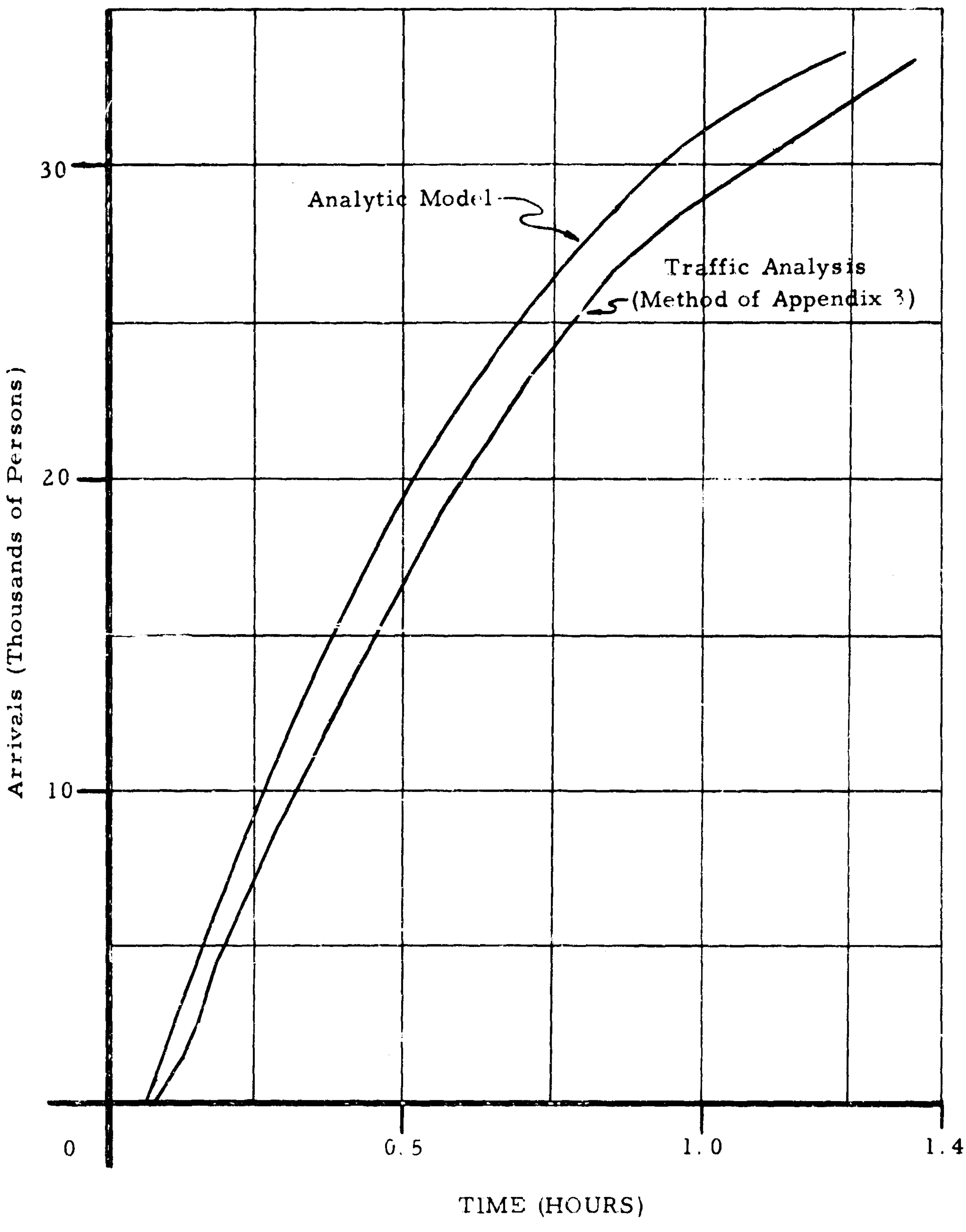

Figure 4. Waterbury, Connecticut, Cumulative Vehicular Arrivals in Shelter Areas vs. Time: 20 MPH One Lane Traffic.

This expression is plotted in Figures 1 and 2 for three values of walking speed for each community. Also plotted is the cumulative walking arrival curve determined by the methods of Appendix for a walking speed of 3 mph. (This latter curve is incomplete in that it does not extend as far in time as the first three. This is because it applies only to those persons who have one mile or less to travel to their shelter areas. Complete comparisons between the two methods are therefore not possible.)

The agreement between the results obtained with the two methods is limited, but without further investigation it is impossible to assert with complete confidence that either is more valid than the other. We do believe, however, that the discrepancies between the two methods are principally due to the inadequacies of the earlier method and that, despite obvious limitations, the method described herein will prove to be far more useful as it is further developed and applied.

One limitation of the model presented here is that, in reality, people will still be arriving from the fringes of the community after the function shows 100% arrivals. This is due to the representation of the community as a circular area (see Section 3.1). Further work is required to determine the precise effects of this idealization and perhaps to investigate the utility of other geometric figures as approximations to the true configuration of the community. Also, the results should be validated against the actual movement of people rather than by comparison with the first crude model as was done here.

Additional work which would greatly increase the utility of the model consists in the determination of the relationship between the parameter a and known characteristics of the community. As long as it is necessary to fit the rate of arrival function to empirical data, the use of this model will require considerable data processing. But when an appropriate value of a can be assumed for any given community, it will be possible to calculate the desired results very quickly or to read them directly from previously prepared charts and graphs.

The curves shown in Figures 1 and 2 show that, at a normal walking speed of about 3 mph, it is possible for everyone to reach shelter in about 45 minutes. They also show that the number of arrivals at any time is much more influenced by delays or lowered walking speeds than by any feasible increase in walking speed. We can therefore tentatively conclude that residents of communities such as Stamford and Waterbury should:

1. <u>know where to go</u>. The distances involved are small enough that everyone can reach shelter before fallout arrives.

2. get moving as quickly as possible. Time lost due to delay cannot be recovered through any feasible increase in walking speed.

3. walk, don't run. The value of additional speed is slight, but the hazards great. Were the population to break into a run, it is likely that fewer people would safely reach shelter because of mob casualties and disrupted traffic flow.

2.2.2 Figures 3 and 4: Cumulative Driving Arrivals

The analytic expression developed to relate vehicular arrivals to time is:

$$D(t) = \frac{P_e}{1 + nb}\left[(1-n)\left\{\frac{t - t_o}{t_m - t_o}\right\}^{b+1} + n(b+1)\left\{\frac{t - t_o}{t_m - t_o}\right\}\right]$$

where P_e is the number of persons who must use vehicles to travel to shelter areas; n is the number of main traffic arteries leading into shelter areas; b is a measure of population groupings in the community; t_o is the time of arrival of the first vehicle; and t_m is the time of arrival of the last vehicle.

Perhaps the outstanding feature of this model is the existence of an explicit relationship between the parameter b and observable, measurable characteristics of the community (see Section 4.3.1). Thus, it is possible to apply this model directly to any community without having to fit the rate of arrival curve to descriptive community data as is currently necessary with the pedestrian traffic model.

The vehicular traffic flow model is applied in Figures 3 and 4 to Stamford and Waterbury for those areas determined to be origins and destinations in Section of the report. The areas and the total numbers of people and shelters involved are therefore not identical to those used in applying the pedestrian traffic flow model. The difference is the exclusion here of all persons residing within origin tracts less than one mile center-to-center distant from their destination. This was done so that the results here would be comparable with the results of the earlier vehicular traffic flow analysis results, which are as close an approximation of reality as is available to us at this time.

The cumulative arrival curves are plotted for a driving speed of 20 mph. The form of both curves compares very favorably with that from our previous, more detailed analysis. The difference between the current and previously calculated function for Waterbury is due primarily to the differences in the estimates of the time of arrival of the last vehicle.

Were there greater precision of significant value in a particular case, it might be worthwhile to employ the method of Appendix to determine the cumulative vehicular arrival function. The difference in the time required, however, in general will give the current model far greater utility. Once the assignments have been made, the main arteries selected, and the values of the constants determined, the entire function can be calculated and plotted by hand in an hour or two; and previously prepared parametric curves could greatly reduce calculation time. The method of Appendix involves over a week of hand computation for a city the size of these two.

No new conclusions have evolved from this effort relative to vehicular traffic flow, both because this model agrees so well with the previous model, and because there has been no opportunity to take advantage of its analytic structure to determine the effects of systematically varying values of the community parameters related to vehicular traffic flow.

2.2.3 Figures 5 and 6: Population Density Curves

The basic postulate of the pedestrian traffic model is that the density of the excess population—those persons who must seek shelter outside of their own tract area—is inversely related to the distance from the shelter area to which they would go. To the extent that this postulate is valid, our confidence in the validity of the model is increased; and to the extent that this postulate is not descriptive of reality, our confidence in the validity of the model is lessened.

In Figures 5 and 6, the postulated relationship is given by a smooth curve fitted to the data by using the values of the parameters computed in Section 5.1 and 5.2. The actual community data are given by the plotted points. Straight lines connecting the points have been drawn as an aid to visualization. Each point plots the distance from the center of an origin tract to the center of the nearest shelter tract against the excess population of that origin tract. Where no one shelter area can accept all of the excess population of a given origin, multiple points appear. While these curves might or might not represent a "best" fit, we feel that the degree of fit is startling, and does give one confidence in the over-all validity of the model.

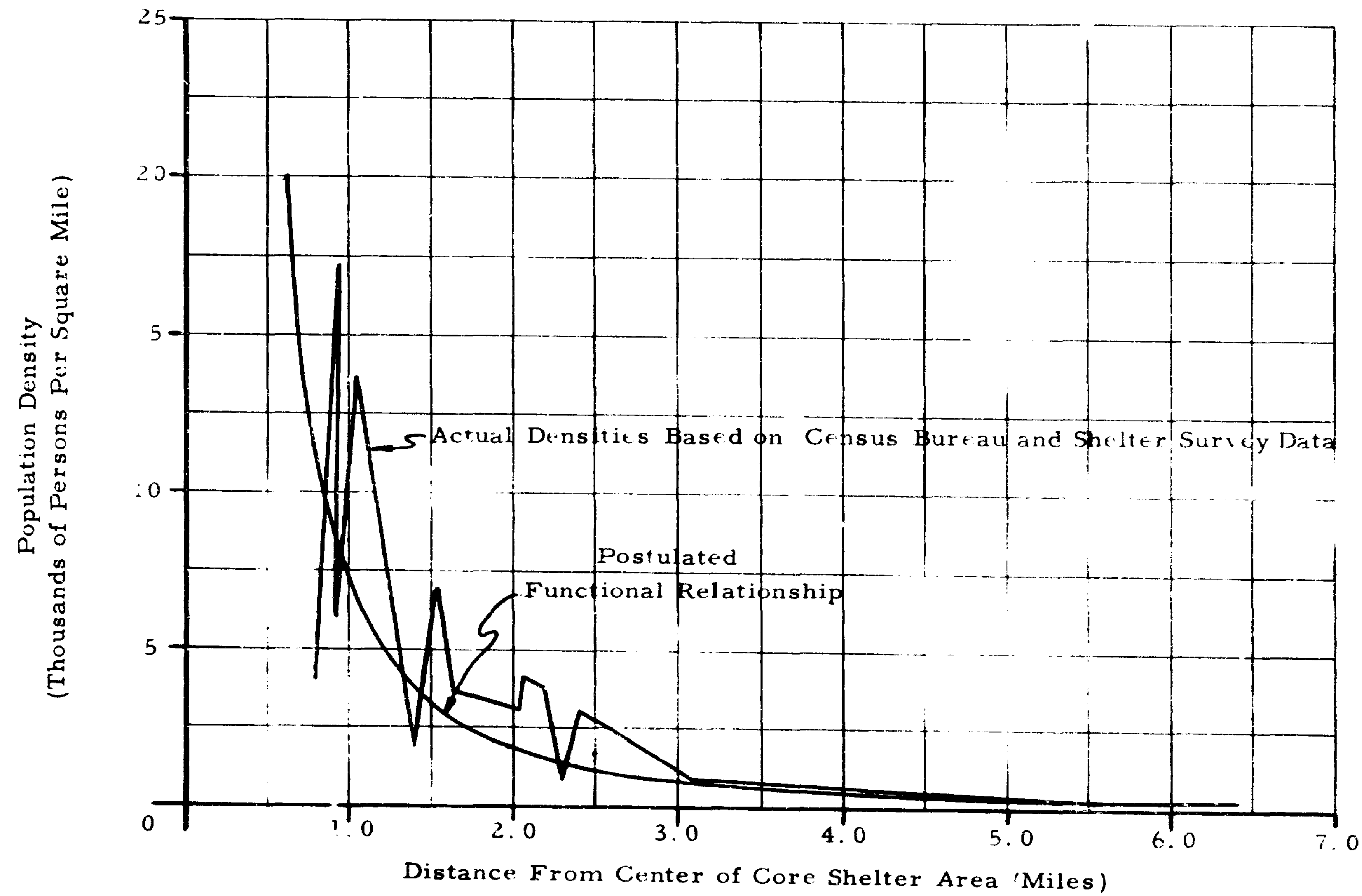

Figure 5. Population Density vs. Distance From Center of Core Shelter Area, Stamford, Connecticut.

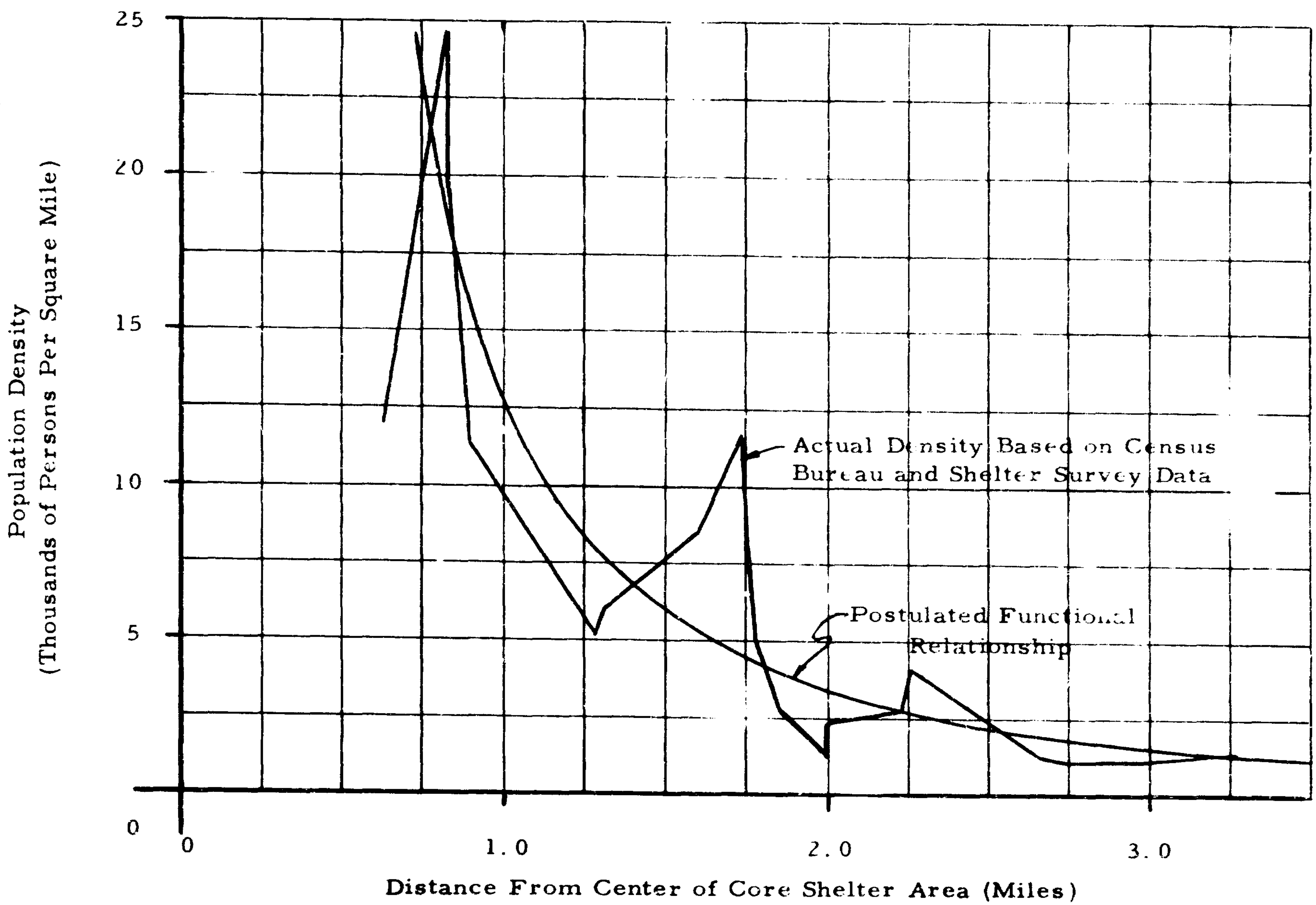

Figure 6. Population Density vs. Distance from Center of Core Shelter Area. Waterbury, Connecticut.

2.2.4 Figures 7 and 8: Rate of Arrival Curves

The basic postulate in the vehicular traffic model is analogous to the population density postulate: we assume regularity in the difference in the number of vehicles originating at each of the centers of excess population. That regularity is described by the postulated mathematical function.

Figures 7 and 8 show the postulated functions for Stamford and Waterbury as continuous, smooth curves, and the actual number of vehicles arriving at the shelter areas as a step function. The step function was generated by plotting the time period over which automobiles would be arriving over each of the main arteries, assuming simultaneous initial arrivals.

Note that, while the curves do indeed fit well, the meaning is not the same here as with the pedestrian traffic model. There, the data fitted were basic community data, and the implication was that the model was a valid one. Here the data fitted are derived using the methods of Appendix 3 , and the implication is that the model developed herein is a valid approximation to those methods. Any inference about the basic validity of this model must be based either on an acceptance of the method of Appendix 3 or upon empirical validation procedures.

3 FORMAL DEVELOPMENT OF THE PEDESTRIAN TRAFFIC MODEL

3.1 The Idealized Community

For computational convenience, the community is represented by a pair of concentric circles with radii $r_1 \leq r_2$. r_1 is selected so that the area of the circle it defines is equal to the area of those tracts with an excess of shelters. r_2 is selected so that it defines an area equal to the area of the entire community. Figures 9 and 10 show the relationship between these idealized areas and the actual areas for Stamford and Waterbury, Connecticut.

Intuitively, it would appear that this geometric approximation is a severe departure from reality, particularly in the case of Stamford. Examination of Figures 5 and 6 in the previous section, however, shows that it leads in both cases to a rather good approximation of that characteristic of the community which we are most interested in approximating—the density of the population at any given distance from the center of the core shelter area.

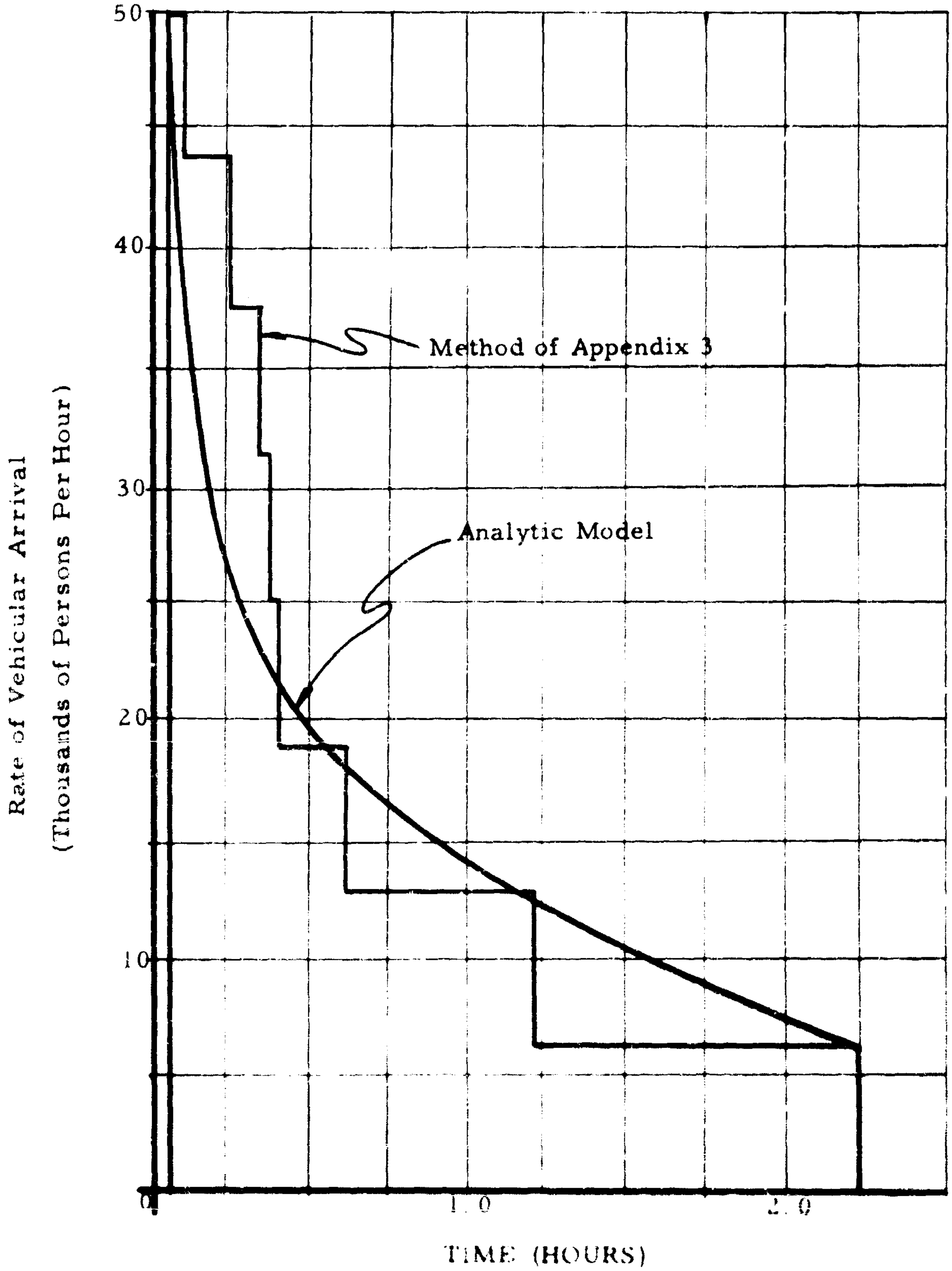

Figure 7. Rate of Vehicular Arrivals vs. Time, Stamford, Connecticut.

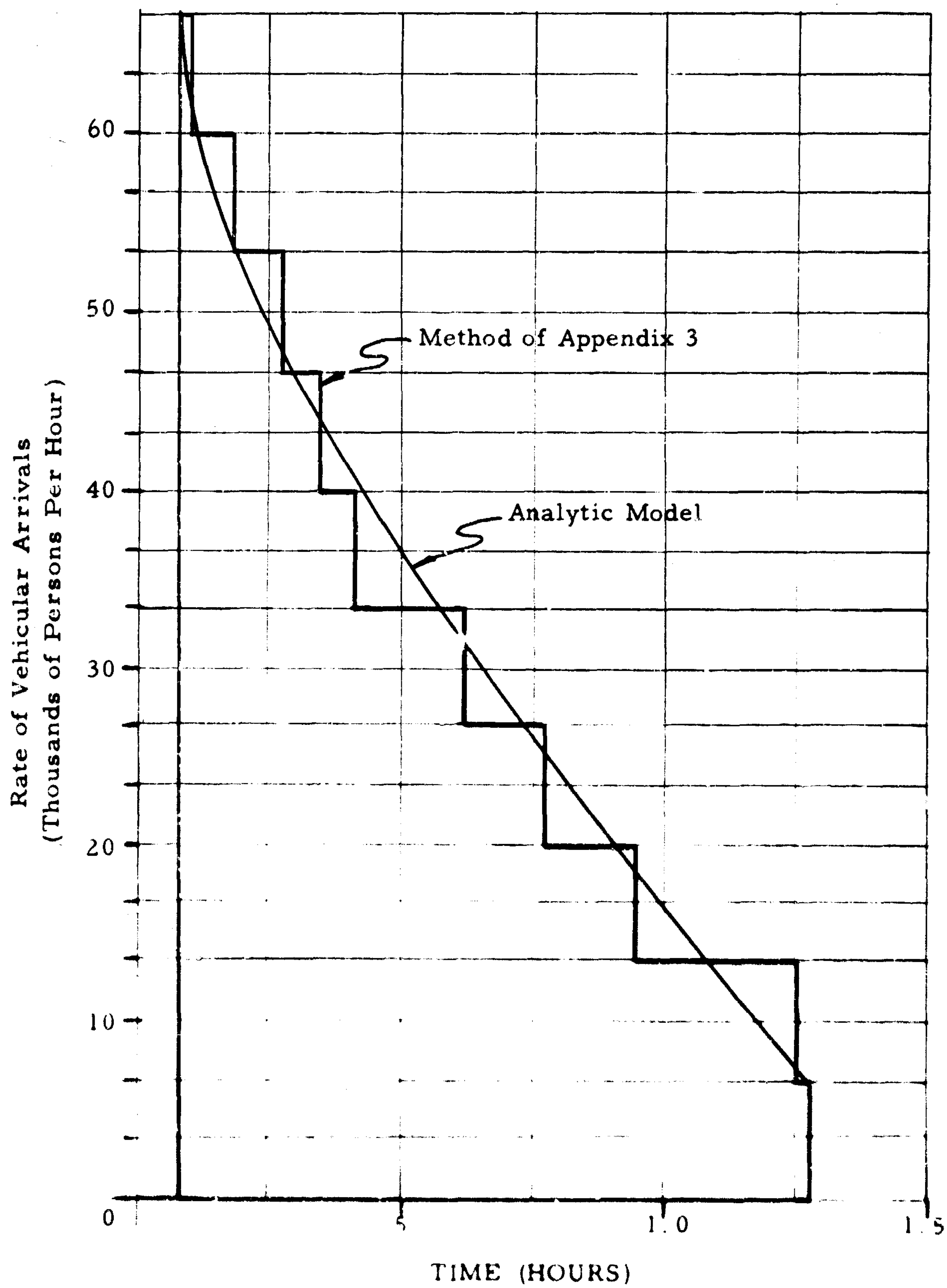

Figure 8. Rate of Vehicular Arrivals vs Time, Waterbury, Connecticut.

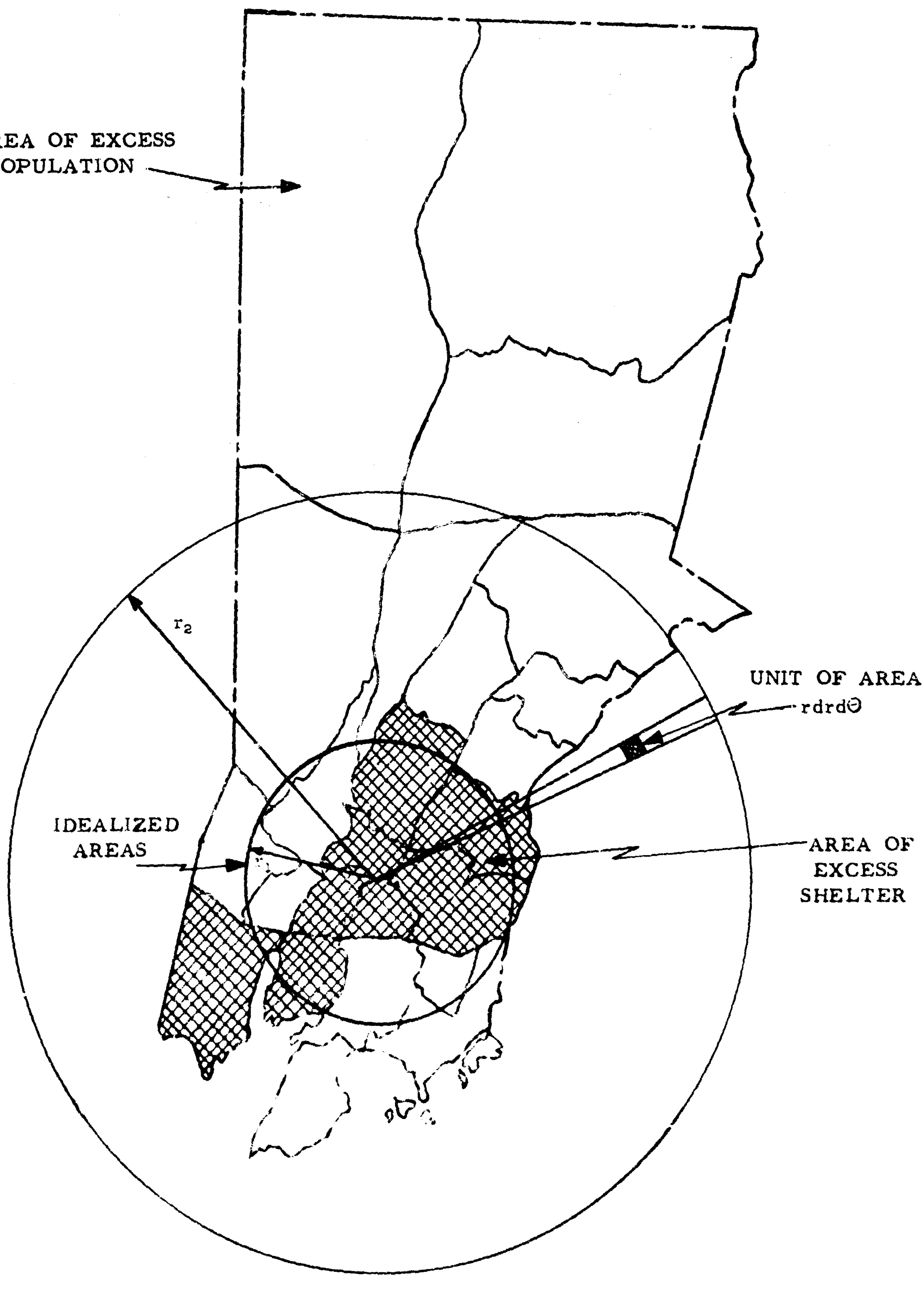

Figure 9. Stamford, Connecticut. Actual and idealized areas of excess population and shelter.

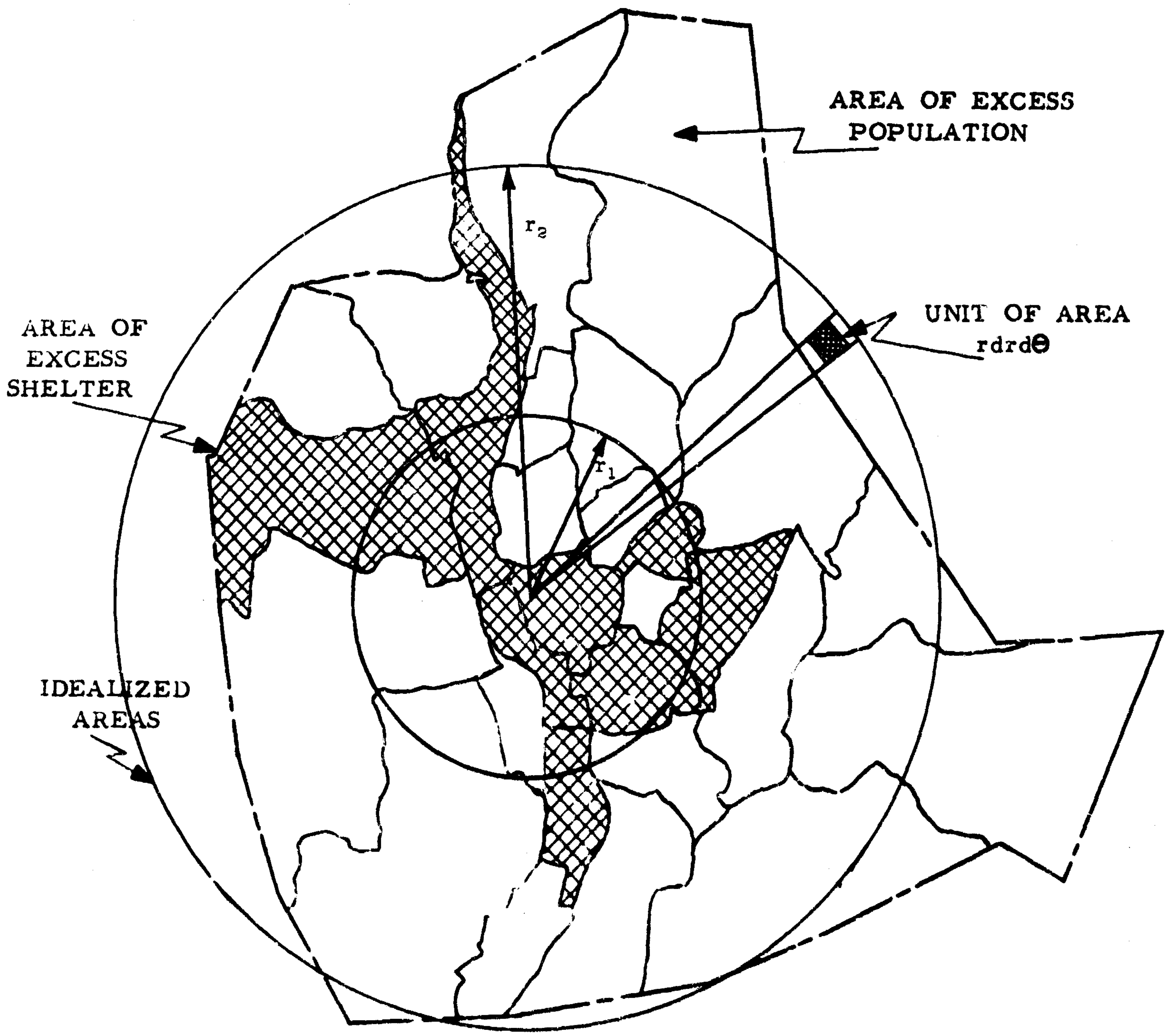

Figure 10. Waterbury, Connecticut. Actual and idealized areas of excess population and shelter.

3.2 Mathematical Development

If we assume that the density of the excess population in the area between r_1 and r_2 decreases with increasing distance from the center of the core shelter area in accordance with

$$\frac{dP}{dA} = \frac{k}{r^a} \tag{1}$$

where k and a are parametric constants, then we can make the following substitution to express equation (1) in polar coordinates

$$dA = r dr d\theta \tag{2}$$

and solve equation (1) for dP

$$dP = kr^{1-a} dr d\theta \tag{3}$$

from which it follows that

$$P = k \iint_0^{2\pi} r^{1-a} dr d\theta \tag{4}$$

which, when integrated, becomes

$$P = 2\pi k \frac{r^{2-a}}{2-a} + C \qquad a \neq 2 \tag{5}$$

Equation (5) describes a family of curves parametric in a. Each of these curves must meet two real-world conditions which permit the evaluation of k and c. The first of these conditions is that none of the excess population is contained within the inner circle which defines the "core shelter area" of the community. That is,

Condition 1: $$P(r_1) = 0$$

from equation (5):

$$P(r_1) = \frac{2\pi k r_1}{2-a} + C$$

Solving for C:

$$C = \frac{-2\pi k r_1}{2-a}$$

and substituting in equation (5):

$$P = \frac{2\pi k}{2-a}\ (r^{2-a} - r_1^{2-a}) \qquad a \neq 2 \qquad (6)$$

Now k is evaluated by considering the second condition—that all of the excess population is contained within the outer circle which defines the limits of the community.

Condition 2: $$P(r_2) = P_e$$

from equation (6):

$$P(r_2) = \frac{2\pi k}{2-a}\ (r_2^{2-a} - r_1^{2-a})$$

Solving for k:

$$k = \frac{P_e(2-a)}{2\pi(r_2^{2-a} - r_1^{2-a})} \qquad (7)$$

and substituting in equation (6):

$$P = P_e\ \frac{(r^{2-a} - r_1^{2-a})}{(r_2^{2-a} - r_1^{2-a})} \qquad a \neq 2 \qquad (8)$$

Equation (7) gives the excess population contained within some circle defined by radius r, where $r_1 \leq r \leq r_2$. We are interested in the time required for this population P to reach the boundary of the core shelter area. Therefore, since

$$t = \frac{r - r_1}{V} \qquad (9)$$

where V is the walking velocity, we substitute

$$r = r_1 + Vt$$

in equation (7), which yields the desired result:

$$W(t) = P_e\left\{\frac{(r_1 + Vt)^{2-a} - r_1^{2-a}}{r_2^{2-a} - r_1^{2-a}}\right\} \qquad a \neq 2 \qquad (10)$$

For the special case $a = 2$, the following result may be derived by taking the limit of equation (10) as a approaches 2:

$$W(t) = P_e \frac{\ln(1 + Vt/r_1)}{\ln(r_2/r_1)} \qquad a = 2 \tag{11}$$

This form of the expression, equation (11), was found to give a good fit to the data for both Stamford and Waterbury, and was therefore used in all calculations in Section 5 of this appendix.

3.3 Evaluating the Parameters

3.3.1 a: The Parametric Constant

The appropriate value of a must be determined by standard curve-fitting procedures, as no relationship between it and other parameters, such as that found for b in Section 4.3.1, has yet been found. Should the value $a = 2$ yield a good fit, as is the case for the two cities discussed in this report, then the computationally simpler form of the final expression given as equation (11) may be used.

3.3.2 r_1: Radius of the Core Shelter Area

The radius of the core shelter area is determined by:

$$r_1 = \sqrt{A_s/\pi} \tag{12}$$

where A_s is the sum of those areas having excess shelter spaces.

3.3.3 r_2: Radius of the Entire Community

The radius of the entire community is given by:

$$r_2 = \sqrt{A_c/\pi} \tag{13}$$

where A_c is the total area of the community, including areas both of excess shelter and of excess population.

3.3.4 V: Walking Velocity

The value of walking velocity is chosen as a planning parameter. In this appendix, walking arrival curves are given for one, three, and five miles per hour walking speed, and compared with the results of our earlier analysis which assumed three miles per hour.

4. FORMAL DEVELOPMENT OF THE VEHICULAR TRAFFIC MODEL

4.1 The Idealized Community

In any community, the bulk of the vehicular traffic is effectively restricted under normal conditions to a relatively few main arteries. We assume that this will be the case also during a civil defense emergency. Figure 11 shows the flow of vehicular shelter-seeking traffic for Stamford, Connecticut, based upon this same assumption in our earlier analysis. Note that, although many roads are used, the traffic rapidly converges on only eight arteries which actually enter the shelter areas.

Figure 12 shows the idealized community configuration upon which the vehicular traffic flow model is based. Each group of tracts having insufficient shelter and using the same main artery to enter a shelter area is conceived of as a single origin reaching the shelter area by a single connecting route. The population of each origin is the sum of the populations of these tracts. All of the tracts with an excess of shelter areas are conceived of as a single "core shelter area." The differences in the length of the different connecting routes are ignored, as they have proved in practice to be negligibly small. Therefore, vehicles from all of the origins will begin to arrive simultaneously in the core shelter area.

4.2 Mathematical Development

Figure 13 shows the discrete function describing the assumed traffic flow. If the vehicles which begin to leave each of the origins simultaneously at time zero and at rate R, the rate of arrival at the destination is zero until time t_o, at which time vehicles begin to arrive simultaneously on all main arteries. If there are n such arteries, Q persons per car, and the vehicle flow rate is R cars per artery per unit time, then the total initial rate of arrival is nQR persons per hour. This rate of arrival holds until all of the people have arrived from the origin of least excess population, at which time the rate of arrival drops discretely to (n - 1)QR. This process continues until time t_m where the last car from the origin with the greatest excess population arrives, and the rate of arrival drops discretely from QR to zero.

The total number of persons involved is given by:

$$P_c = QR\,\Sigma(t_i - t_o) \qquad i = 1, 2, \ldots, n \qquad (14)$$

where t_i is the duration of traffic flow on the i-th main artery. In the special case where the traffic flow on succeeding arteries is exhausted at regular

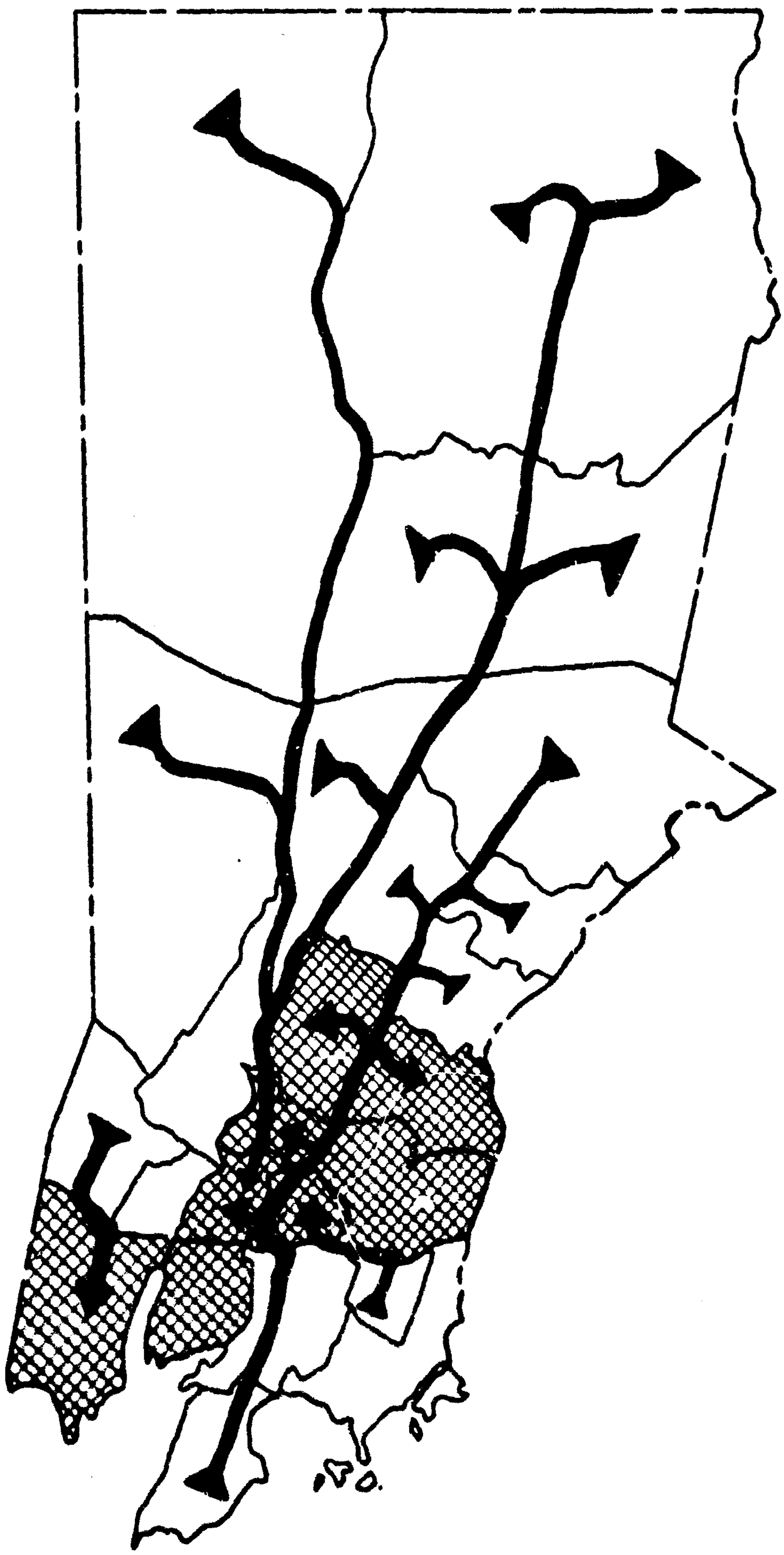

Figure 11. Stamford, Connecticut Actual Vehicular Traffic Flow.

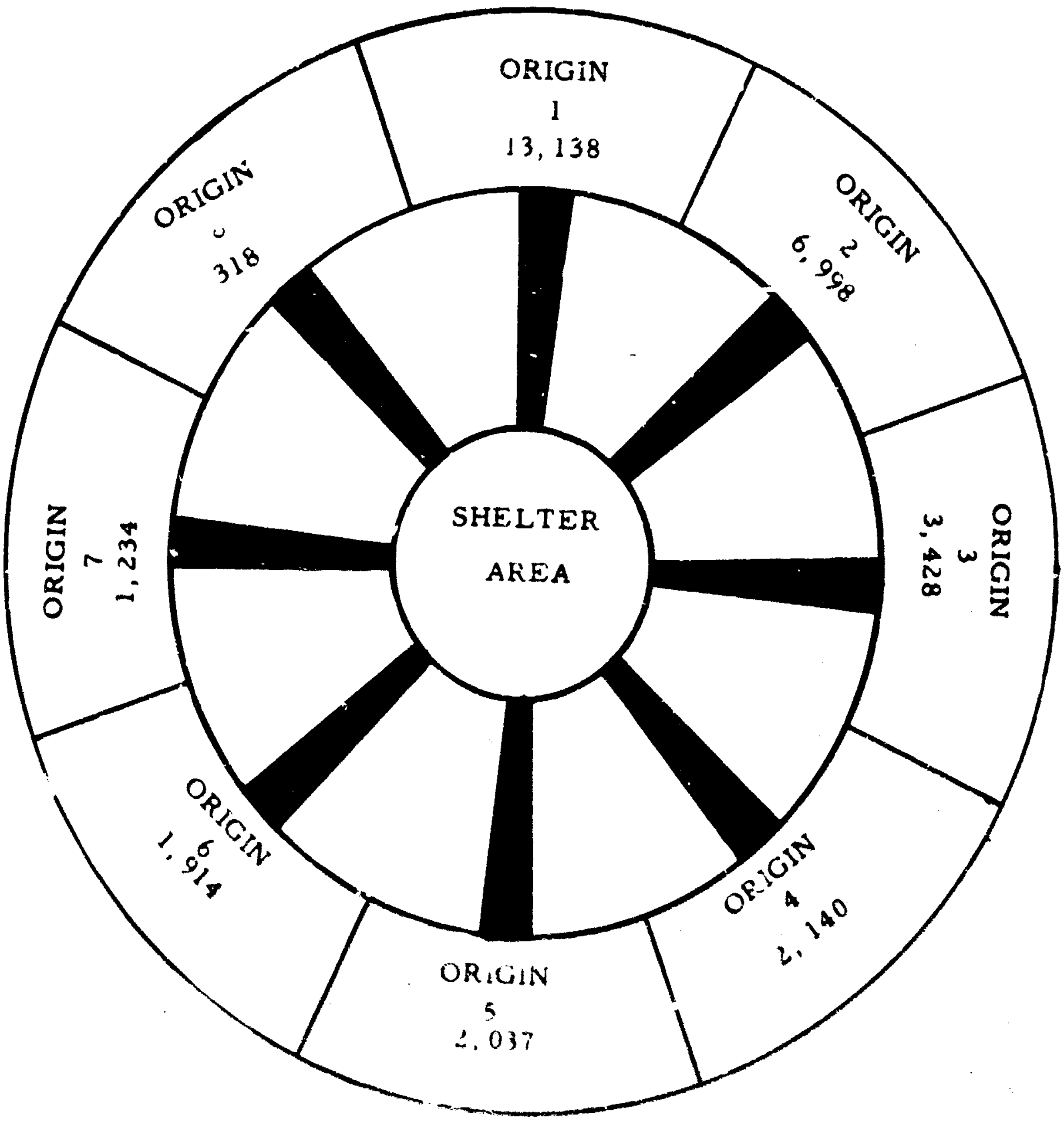

Figure 12. Stamford, Connecticut Idealized Area (Vehicular Traffic).

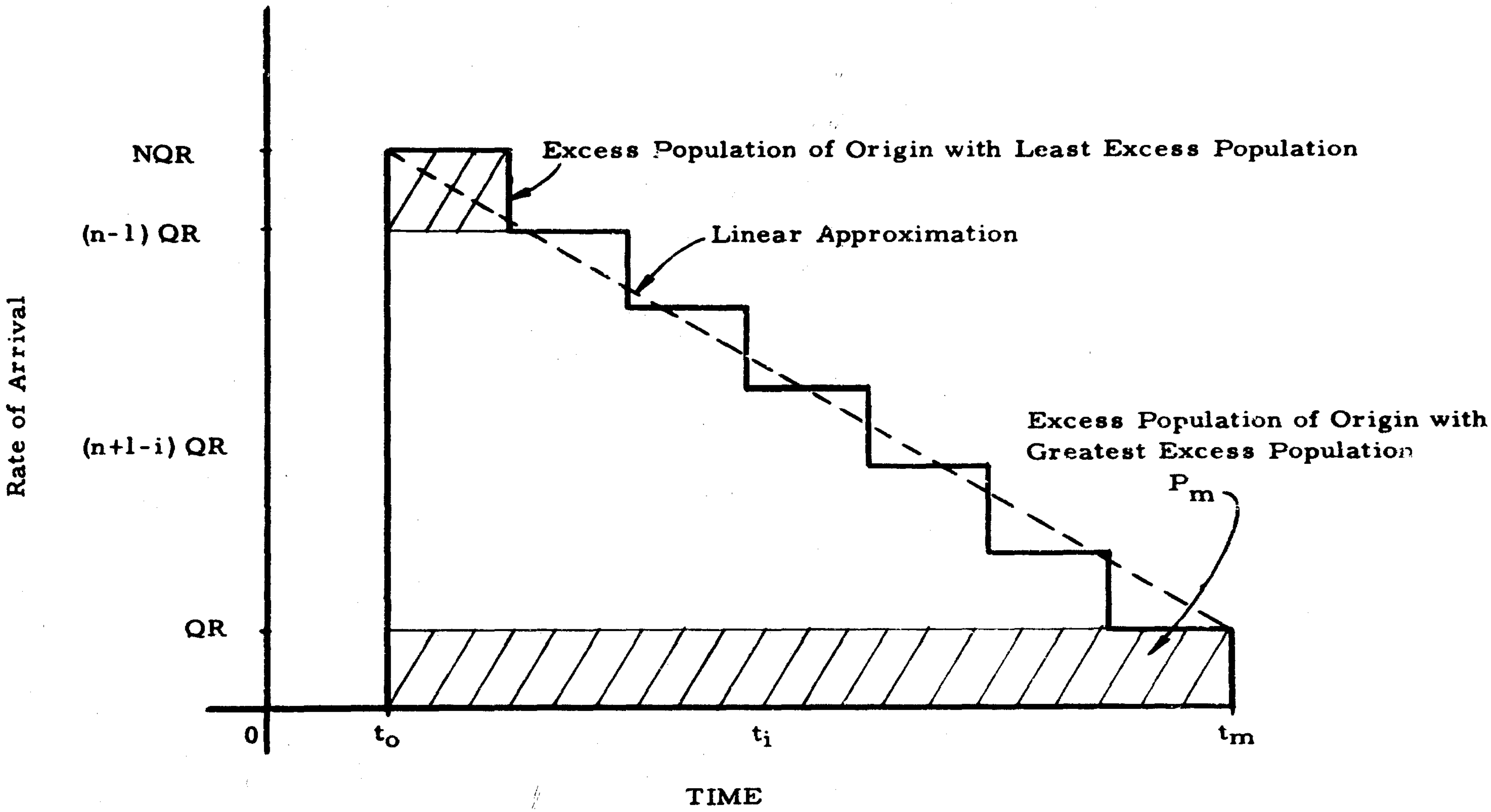

Figure 13. Discrete Rate of Arrival Function

intervals, traffic continues to flow on each artery for exactly $(1/n)(t_m - t_o)$ time units longer than on the preceding one before becoming exhausted. Therefore

$$t_i - t_o = \frac{(t_m - t_o)}{n} (n - j) \qquad j = n + 1 - i \tag{15}$$

and

$$P_e = QR \frac{(t_m - t_o)}{n} \sum_{j=1}^{n} (n - j)$$

$$P_e = QR (t_m - t_o) \frac{(1 + n)}{2} \tag{16}$$

The discrete rate of arrival function discussed above can be approximated by the continuous function (see Figure 14):

$$D'(t) = QR \left[(1 - n) \left\{\frac{t - t_o}{t_m}\right\}^b + n\right] \qquad t_o \leq t \leq t_m \tag{17}$$

where

- t = time
- Q = number of persons per vehicle
- R = vehicle arrival rate in cars per artery per unit time
- n = number of arteries leading into shelter areas
- t_o = time of arrival of first vehicle
- t_m = time of arrival of last vehicle
- b = a parametric constant

which has the following properties

$$D'(t_o) = nQR \tag{18.1}$$

$$D'(t_m) = QR \tag{18.2}$$

$$D(t_m) = P_e \tag{18.3}$$

These properties may be verified by inspection. The quantity $(t - t_o)/(t_m - t_o)$ will vary from zero to one as t varies from t_o to t_m. When it is equal to zero, the value of the entire expression is nQR; when it is equal to one, the value of the entire expression is QR. The third property (18.3) is used to fit the curve to the data for any given community.

$$D'(t) = QR\left[(1-n)\left\{\frac{t-t_o}{t_m-t_o}\right\}^b + n\right]; \quad t_o \leq t \leq t_m \qquad 0 \leq b < \infty$$

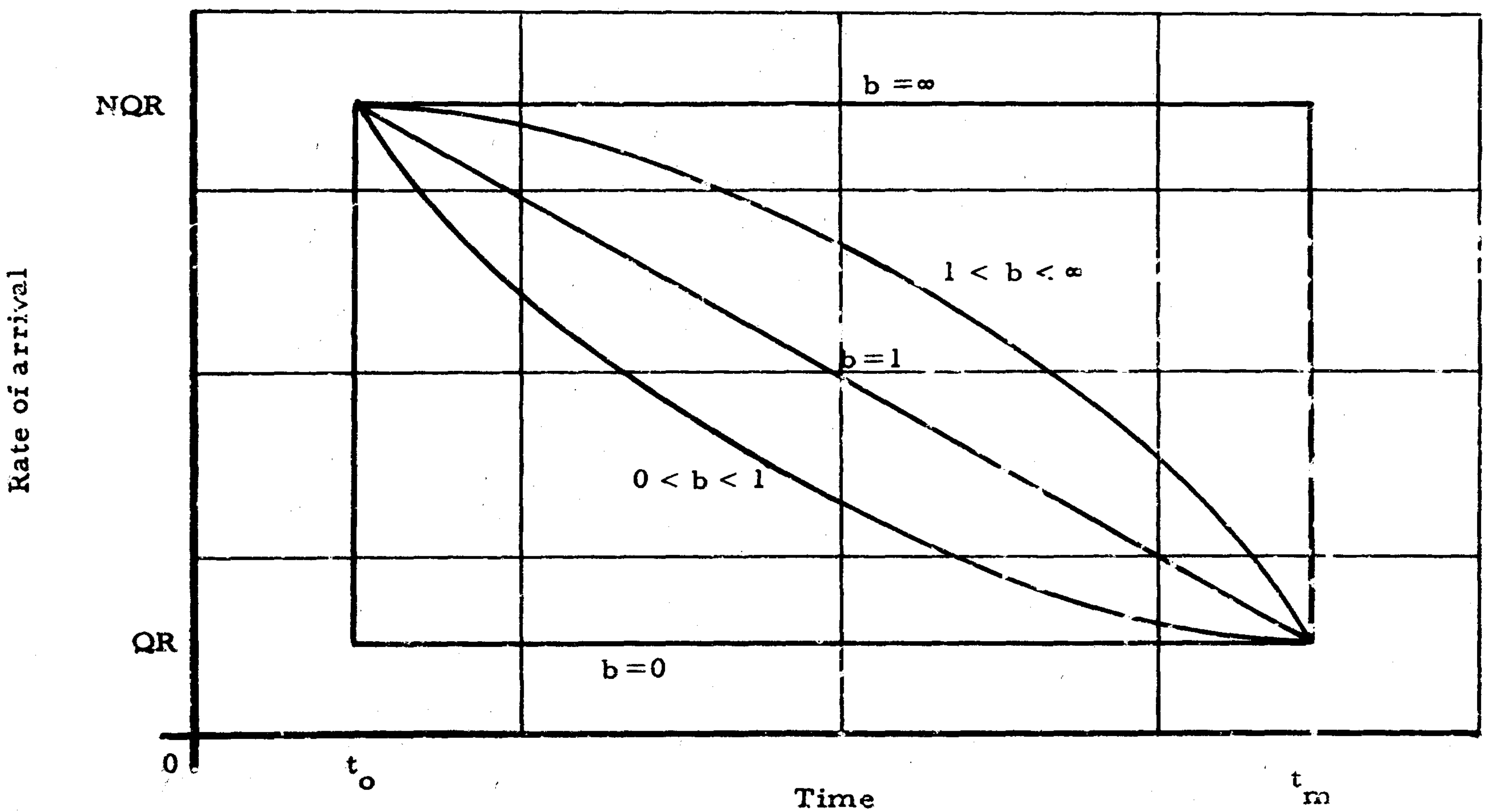

Figure 14. Continuous rate of arrival showing effect of various values of Parameter b.

Examination of the geometry of Figure 13 will make clear the relationship between $D(t)$ and P_e. In the special case where the traffic flow on succeeding arteries is exhausted at regular intervals, the parameter b is equal to unity and the area under the linear approximation is equal to $QR(t_m - t_o)(1+n)/2$ which is in agreement with equation (16).

In general, however, the function is not linear and it is necessary to use empirical data to calculate the value of b (see Sections 5.3 and 5.4 of this appendix. When b is less than unity, the function is concave upward; when b is greater than unity, the function is concave downward. These values of the parameter correspond respectively to the real-world cases where the total number of vehicles is less evenly distributed and more evenly distributed among the several main arteries.

The cumulative arrival function, which is what we are really interested in, is obtained by integrating $D'(t)\,dt$ subject to the initial condition expressed in (18.3).

$$D(t) = QR\int_{t_o}^{t}\left[(1-n)\left\{\frac{t-t_o}{t_m-t_o}\right\}^b + n\right]dt \qquad (19)$$

First the appropriate substitutions are made in order to integrate with respect to $\left\{\frac{t-t_o}{t_m-t_o}\right\}$: $\dfrac{t-t_o}{t_m-t_o}$

$$D(t) = QR(t_m-t_o)\int_{0}\left[(1-n)\left\{\frac{t-t_o}{t_m-t_o}\right\}^b + n\right]\frac{dt}{t_m-t_o} \qquad (20)$$

and the expression is integrated:

$$D(t) = \frac{QR(t_m-t_o)}{1+b}\left[(1-n)\left\{\frac{t-t_o}{t_m-t_o}\right\}^{b+1} + n(b+1)\left\{\frac{t-t_o}{t_m-t_o}\right\}\right] \qquad (21)$$

then property (18.3) is employed:

$$P_e = D(t_m) = QR(t_m-t_o)\frac{(1+nb)}{(1+b)} \qquad (22)$$

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to obtain the final expression

$$D(t) = \frac{P_e}{1+nb}\left[(1-n)\left\{\frac{t-t_o}{t_m-t_o}\right\}^{b+1} + n(b+1)\left\{\frac{t-t_o}{t_m-t_o}\right\}\right] \qquad (23)$$

4.3 Evaluating the Parameters

4.3.1 b: The Parametric Constant

The value of b is derived by observing in Figure 13 that

$$P_m = QR(t_m - t_o) \qquad (24)$$

where P_m is the excess population of the area of greatest excess population and by substituting P_m for its equivalent in equation (22):

$$P_e = P_m \frac{(1+nb)}{1+b} \qquad (25)$$

Solving equation (25) for b, the result is:

$$b = \frac{P_m - P_e}{P_e - nP_m} \qquad (26)$$

4.3.2 Q: The Number of Persons Per Vehicle

Q is based on Census Bureau or other statistics. For automobiles, Q obviously will fall somewhere between one and eight; the more likely value being about four. In Section 5.2, Q is found by

$$Q = \frac{P_{mt}}{H} \qquad (27)$$

where P_{mt} is the total population of the origin with the greatest excess population, and H is the number of households in that origin having one or more cars.

4.3.3 R: Vehicle Arrival Rate

The vehicle arrival rate for each artery is based upon an estimate of the capacity of the roads leading into the shelter areas. The arrival rate for single lane, 20 mph traffic (1650 vph) derived earlier is used here.

4.3.4 n: Number Arteries Leading Into Shelter Area

The number of arteries leading into the shelter area is determined by studying the area, and deciding which roads as a matter of policy will be designated main arteries. In many cases, local geography and street lay-out will limit the choices available; in all cases, it will be necessary to consider traffic and other planning activities.

4.3.5 t_o: Time of Arrival of First Vehicle

t_o is determined by the relationship:

$$t_o = \frac{d_{min}}{S} \tag{28}$$

where d_{min} is the least distance from an origin to a destination area and S is the road speed of the vehicles.

4.3.6 t_m: Time of Arrival of Last Vehicle

t_m is determined by the relationship:

$$t_m = d_m + \frac{P_m}{QR} \tag{29}$$

where d_m is the distance from the origin of greatest excess population and P_m is the excess population of that origin.

5. COMPUTATIONS

5.1 Stamford Pedestrian Traffic

The total shelter area in Stamford, Connecticut, comprising eight of the 24 census tracts in that city, is 4.82 square miles. The area of the entire community is 37.53 square miles.[1] Therefore using equations (12) and (13):

[1] Areas were obtained by direct measurement of the maps appearing the PHC series publications of the 1960 Census of the U.S. and may differ slightly from those published elsewhere. This does not affect the validity of the results obtained.

$$r_1 = \sqrt{4.827/\pi} = 1.235 \text{ miles}$$

$$r_2 = \sqrt{37.53/\pi} = 3.455 \text{ miles}$$

Normally, it is not necessary to compute the value of k. It is done here in order that the relationship assumed in equation (1) can be plotted to demonstrate the degree of fit realized by assuming the value of a equal to 2. The limit of equation (7) as a approaches 2 (the value assumed herein), is given by:

$$k = \frac{P_e}{2\pi \ln(r_1/r_2)} \qquad a = 2 \qquad (30)$$

The excess population in Stamford numbers 44, 397. Therefore

$$k = \frac{44397}{2\pi(1.03)} \cong 6,850$$

The expression for the rate of arrival is therefore

$$dP = 6850r^{-2}$$

and, using equation (11), the expression for the cumulative number of arrivals is

$$P = 43000\ln(1 + \frac{Vt}{1.235})$$

The rate of arrival curve appears as Figure 5 on page 4-11. The cumulative arrival curves are plotted for three values of V in Figure 1 on page 4-4.

5.2 Waterbury Pedestrian Traffic

The shelter area in Waterbury is 5.46 square miles, and the area of the entire community is 30.12 square miles.[1] Therefore

$$r_1 = \sqrt{5.46/\pi} = 1.32 \text{ miles}$$

$$r_2 = \sqrt{30.12/\pi} = 3.10 \text{ miles}$$

[1]See footnote on previous page.

Again assuming $a = 2$, using equation (30) and substituting the $P_e = 68,042$,

$$k = \frac{68042}{2\pi(.851)} \cong 12,700$$

$$dP = 12,700r^{-a}$$

and, using equation (11),

$$P = 80,000\ln(1 + \frac{Vt}{1.32})$$

These expressions are plotted in Figure 6, page 4-12, and in Figure 2, page 4-5.

5.3 <u>Stamford Vehicular Traffic</u>

From the 1960 Census of the United States, the National Shelter Survey Phase II results, and a detailed map obtained from the city engineering department, the following values have been obtained:[1]

P_e = 31,307

P_m = 13,138

P_{mt} = 13,569

d_{min} = .130 miles

d_m = .306 miles

n = 8

H = 3,586

[1]Implicit in these figures is an assignment of persons from origins to certain destinations and to movement on foot or by vehicle. The assignment made here is the same as in our earlier analysis so that comparisons can be made between the two approaches. This is why the value of P_e here differs from th[illegible] used in the application of the walking model.

From these, the following were calculated:

$$b = \frac{P_m - P_e}{P_e - nP_m} = \frac{13,138 - 31,207}{31,207 - 8(13,138)} = .2445$$

$$t_o = \frac{d_{min}}{S} = \frac{1.30}{20} = .065 \text{ Hrs.}$$

$$t_m = \frac{d_m}{S} + \frac{P_m}{QR} = \frac{3.06}{20} + \frac{13,138}{(3.7839)(1650)} = 2.257 \text{ Hrs.}$$

$$Q = \frac{P_{mt}}{H} = \frac{13,569}{3,586} = 3.7839$$

The rate of arrival function for Stamford is determined by substituting the above in equation (17):

$$D_S'(t) = (3.7839)(1650)\left[(1-8)\left\{\frac{t-.065}{2.257-.065}\right\}^{.2445} + 8\right] \qquad .065 \le t \le 2.257$$

$$D_S'(t) = 6243\left[-7\left\{\frac{t-.065}{2.192}\right\}^{.2445} + 8\right] \qquad .065t \le 2.257$$

and the cumulative arrival function is similarly determined by substitution in equation (23):

$$D_S(t) = 31,207\,\frac{\left[(1-8)\left\{\frac{t-.065}{2.257-.065}\right\}^{1+.2445} + 8(1+.2445)\left\{\frac{t-.065}{2.257-.065}\right\}\right]}{1+8(.2445)}$$

$$.065t \le 2.257$$

$$D_S(t) = 1056\left[7\left\{\frac{t-.065}{2.192}\right\}^{1.2445} + .995\left\{\frac{t-.065}{2.192}\right\}\right] \qquad .065 \le t \le 2.257$$

These curves—the rate of arrival and the cumulative arrivals—appear as Figure 7, page 4-14, and Figure 3, page 4-6.

5.4 Waterbury Vehicular Traffic

From the same sources as for Stamford, the following data were obtained:[1]

$$P_e = 33,401$$

$$P_m = 7,698$$

$$P_{mt} = 8,482$$

$$d_{min} = 1.30 \text{ miles}$$

$$d_m = 1.70 \text{ miles}$$

$$n = 10$$

$$H = 2,088$$

From these, the following were calculated:

$$b = \frac{P_m - P_e}{P_e - nP_m} = \frac{7698 - 33401}{33401 - 10(7698)} = .5898$$

$$t_o = \frac{d_{min}}{S} = \frac{1.30}{20} = .065 \text{ Hrs.}$$

$$Q = \frac{P_{mt}}{H} = \frac{8482}{2088} = 4.062$$

$$t_m = \frac{d_m}{S} + \frac{P_m}{QR} = \frac{1.70}{20} + \frac{7698}{(4.062)(1650)} = 1.234 \text{ Hrs.}$$

[1]See footnote on Page 4-32.

The rate of arrival curve for Waterbury is, by substitution, in equation (17):

$$D'_W(t) = (4.062)(1650)\left[(1-10)\left\{\frac{t-.065}{1.234-.065}\right\}^{.5898} + 10\right] \qquad .065 < t < 1.234$$

$$D'_W(t) = 6702\left[9\left\{\frac{t-.065}{1.168}\right\}^{.5898} + 10\right] \qquad .065 < t < 1.234$$

and the cumulative arrival function is, by substitution, equation (23):

$$D_W(t) = 33401\ \frac{\left[(1-10)\left\{\frac{t-.065}{1.234-.065}\right\}^{1+.5898} + 10\left(1+.5898\left\{\frac{t-.065}{1.234-.065}\right\}\right)\right]}{1+10(.5898)}$$

$$.065 < t < 1.234$$

$$D_W(t) = 4842\left[9\left\{\frac{t-.065}{1.169}\right\}^{1.5898} + 15.898\left\{\frac{t-.065}{1.169}\right\}\right] \qquad .065 < t < 1.234$$

These curves appear as Figure 8, page 4-15, and as Figure 4, page 4-9.

APPENDIX 5

Model to Predict Number of Tract Residents Sheltered Within Tract as a Function of Elapsed Time From When Residents Leave Homes

This model will be developed with the aid of an illustration. The illustration is for census tract #14 of the City of Stamford, Connecticut. The basic data required by the model is as follows:

Residents to be sheltered in the tract:	4781
Number of shelters in tract:	10
Tract area:	.642 mi.^{2}
Population density of tract:	7450 people per mi.^{2}

It is assumed that the number of residents per unit area is constant throughout the tract, and it is also assumed that the shelters are so spaced that an area and hence a fixed number of residents can be associated with each shelter. For the present illustration the sub-area associated with each shelter is .0642 sq. miles, and the number of residents in this sub-area is 478.1. It is also assumed that the residents within a shelter sub-area will always proceed to that shelter first and, if it is full, will then proceed to the next nearest shelter.

It is convenient to rank the shelters in order of their capacity as shown in Figure 1. When the warning is received by the residents, they will proceed to their nearest shelter. Thus, 478 residents will "visit" each of the ten tract shelters and, as can be seen from Figure 1, some of the residents will find a shelter space available while others will not. Table 1 shows that 3,474 residents can expect to find shelter, while 1,307 will have to visit at least one more shelter to find space. The situation faced by the 1,307 unsheltered residents is that three shelters are not full and seven shelters are full. However, each resident has visited a full shelter and presumably is aware of its location so will proceed to one of the remaining nine shelters. Because six of the remaining shelters are full, 6/9 of the unsheltered residents, i.e., 871, will be unsheltered after these second visits to a shelter. Three-ninths of the unsheltered residents, i.e., 436, will visit shelters which are not completely full. It is assumed that these 436 residents will divide evenly among the three partially full shelters, or 145 residents per shelter. Returning again to Figure 1, it can be seen that 145 residents per

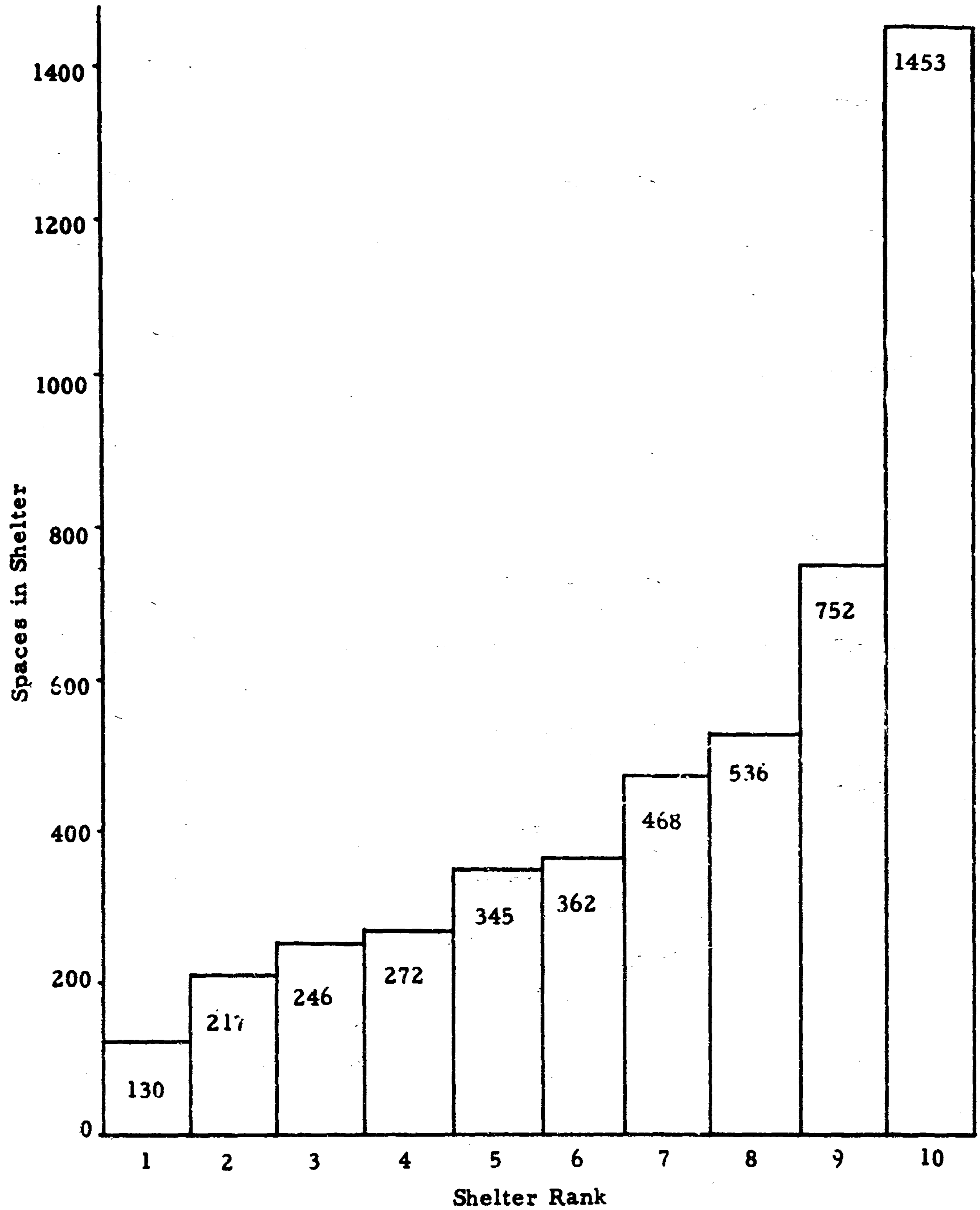

Figure 1. Rank of Shelters in Order of Capacity--Tract No. 14.

partially filled shelter fills up one more shelter (Rank 8) and leaves 87 residents more still unsheltered. Thus, there are 871 plus 87 or 958 residents that must make a third visit to a shelter. The situation faced by these 958 residents is two shelters partially filled and 8 shelters completely full. However, each resident has now visited two full shelters and presumably will not return to them. Thus, each resident now has a "population" of eight shelters to visit, two of which are partially full and six of which are completely full. Again, it is assumed that 6/8 (i. e., 718) of the residents visit full shelters and must make a fourth visit. 240 residents visit shelters which are partially empty, or 120 residents per shelter. It can be seen from Figure 1 that 120 residents do not fill up either of the shelters, hence all 240 are now sheltered, leaving 718 to make a fourth visit.

The procedure is now continued until all residents are sheltered. The number of residents sheltered after each visit in contained in Table 1, which follows:

Table 1

Cumulative Residents Sheltered

Shelter Visits	Residents Unsheltered Before Shelter Visit	Residents Sheltered After Visit	Residents Requiring Another Shelter Visit	Shelters Partially Full at end of Visit	Shelters Completely Full at end of Visit
1	4781	3474	1307	3	7
2	1307	349	958	2	8
3	958	240	718	2	8
4	718	129	589	1	9
5	589	98	491	1	9
6	491	98	393	1	9
7	393	98	295	1	9
8	295	98	197	1	9
9	197	98	99	1	9
10	99	99	0	0	10

The next step in the model is to compute the number of residents sheltered as a function of time. For purposes of this computation, it is assumed that the sub-area associated with a shelter is in the form of a circle. Each sub-area contains .0642 mi.^{2}, which corresponds to a circle of .143-mile radius. Returning now to Figure 1, it can be seen that 130 residents visit each shelter before

the first shelter is full. From the population density of the tract, it is found that these residents come from an area of .0174 sq. miles surrounding the shelter. The radius of this area is .0745 miles. Thus, the first of this group of 130 residents travels zero miles to the shelter while the last travels .0745 miles. At a speed of 3 mph, elapsed time for the last of the group to be sheltered is .0247 hours.[1] Because there are ten shelters, 1300 residents are sheltered in the time period from zero to .0247 hours after warning. A second shelter is filled after 87 more residents visit each shelter. Again from population density, it is found that this group of residents may have to walk as far as .0962 miles to get to the shelters. There are now nine shelters which accept these residents for a total of 783 people. At 3 mph, the time period during which this group of 783 people is sheltered extends from .0247 hours to .0321 hours. Thus, at the end of .032 hours the total number of residents sheltered is 1300 + 783, or 2083. This process is continued until the number of residents finding shelter on the first shelter visit and the time period during which they find shelter are determined.

It is now necessary to determine the time required to find shelter for those residents who are not sheltered on their first visit to a shelter. There are 478 residents associated with each shelter in the census tract and, as was noted earlier, one shelter is filled by 130 residents leaving 348 residents to visit this shelter and then proceed to the next nearest shelter. The time required for the shelter to fill is .0247 hours, and .0230 hours later (i.e., .0477 hours. from departing homes) each resident has visited the shelter with which he has been associated. The time interval of .0230 hours also represents the interval during which residents who have not found space in the first shelter visited are arriving at a second shelter.

The minimum distance between shelters in this illustration is .286 miles, therefore, the minimum distance traversed in visiting the first two shelters is .360 miles and the maximum distance travelled is .429 miles. At 3 mph, these distances correspond to .120 hours and .143 hours respectively, which means that all residents who find shelter space on their second visit to the shelter are sheltered in the time interval between .120 hours and .143 hours after departing their homes. From Table 1 it can be seen that 349 residents are sheltered during this time period. The time period during which residents find shelter space during their third visit to a shelter is determined by adding the transit time between shelters (i.e., .095 hours) to the time of arrival at the second shelter. Thus, the time interval during which residents are sheltered as a result of the third visit to shelters is .120 + .095 = .215 hours and

[1]Assuming no bottleneck at the shelter entrance.

.143 + .095 = .238 hours respectively. This procedure is continued until the time intervals have been established for all residents seeking shelter within the tract. The results of applying the resident sheltering model to Tract 14 are contained in Table 2, which follows.

Table 2

Residents Sheltered as a Function of Time for Census Tract 14 of the City of Stamford, Connecticut

Residents Sheltered	Time from Departing Homes (hrs.)
1300	.025
2083	.032
2315	.034
2497	.036
2935	.040
3020	.041
3444	.047
3474	.048
3474	.120
3823	.143
3823	.215
4063	.238
4063	.310
4192	.333
4192	.405
4290	.428
4290	.500
4388	.528
4388	.595
4486	.618
4486	.690
4584	.713
4584	.785
4682	.808
4682	.880
4781	.903

APPENDIX 6

Model to predict the number of nontract residents sheltered within a tract having an excess of shelter spaces, as a function of elapsed time from when nontract residents arrive.

Appendices 3 and 5 contain two models which represent respectively:

- The process by which people who cannot find shelter within their own tract are assigned to and arrive at tracts having shelters with excess capacity. The end-product of this model is the arrival of people at the boundaries of tracts having excess capacity as a function of time.

- The process by which people who are located within a census tract enter shelters within that same tract. The end-product of this model is the number of people sheltered as a function of time.

The model described here picks up where the above two models leave off. It predicts the number of nontract residents sheltered within a tract having an excess of shelter spaces, as a function of time. An illustration will be used as an aid in developing the model. The illustration is for census tract #12 in the City of Stamford, Connecticut.

I. REQUIRED DATA

The following data are a prerequisite for use of the model.

1. The number of shelters in the tract being considered, having possible protection factors of 100 or better, and the remaining capacity of each shelter after residents of that tract have taken shelter. Table 1 contains this information for tract #12, City of Stamford, Connecticut. The number of shelters to be considered plus the "Total Shelter Spaces Before Tract Residents Take Shelter" is taken from the National Fallout Shelter Survey, Phase 2, under the classification "Total Possible Shelter Spaces, with protection factor 100 or better." "Shelter Capacity After 3,324 Tract #12 Residents Take Shelter" is determined using the model described in Appendix 4.

Table 1

Shelter Capacity in Tract #12

Facility Number	Total Shelter Spaces Before Tract Residents Take Shelter	Shelter Capacity After 3,324 Tract #12 Residents Take Shelter
1	353	0
2	695	0
3	2,625	1,487
4	3,043	1,905

2. The area of shelter tract: For tract #12 the area is 1.055 square miles. This information can be obtained in a number of ways. In this case a planometer was used on a 1/9600 scale map of Stamford.

3. The number of people from other tracts seeking shelter in the tract being considered, and their time of arrival and rate of arrival at the tract: For tract #12 this information is listed in Table 2.

Table 2

Number and Arrival Rate of Nontract #12 Residents Arriving in Tract #12

Number of Nontract #12 Residents Arriving in Tract #12	Time of Arrival (minutes)	Time Period (minutes)	Rate of Arrival (People/Minute)
1355	0 - 22.8	22.8	59.43
0	22.8 - 41.4	18.6	0
2037	41.4 - 58.1	16.7	121.63

The data in this table are determined by using the model described in Appendix 3.

II. THE USE OF ASSUMPTIONS IN THE MODEL

This model is based on a number of assumptions. These assumptions are listed below along with appropriate illustrations of their use in predicting the sheltering of a nonresident population in tract #12 shelters.

1. The shelters within a tract are assumed to be uniformly distributed within that tract and to service equal areas such that the sum of these areas equals the area of the entire tract. Under these conditions the distance between shelters can be represented by equation (1):

$$d = \sqrt{\frac{A}{N}} \tag{1}$$

where: d = the distance between adjacent shelters in miles.

A = the area of the tract in square miles.

N = the number of shelters having possible protection factors of 100 or better.

In this case, A = 1.1 square miles (page 6-2), and N = 4 shelters (page 6-2), therefore, for tract #12, d = .52 miles.

2. It is assumed that each shelter within a tract has an equal likelihood of being the first shelter a new arrival finds, and that if this shelter is full, safety is sought in the next nearest shelter. All movements are straight-line distances from one shelter to another. This process continues until a person arrives at a shelter having excess shelter capacity.

3. All movement toward shelters within a tract is assumed to be on foot, at the rate of 3 miles per hour or .05 miles per minute.

4. People arrive at a tract, in search of shelter, either on foot or by automobile. If they arrive on foot, measurement of the distance they travel to their "first" shelter begins as they cross into the tract. If they arrive by car, the distance to their "first" shelter is measured from wherever they park their car to the nearest shelter. In both cases this initial distance to a "first" shelter can be represented by equation (2):

$$R = .381\,d \tag{2}$$

where: R = mean distance to the first shelter in miles.

d = distance between adjacent shelters in miles (equation (1)).

In this case, d = .52 miles; therefore, R for tract #12 =.20 miles.

5. All shelters with excess capacity are assumed to fill up at the same rate. On the basis of this assumption it is possible to calculate the incremental number of persons who must take shelter before each additional shelter is filled. For the example used here this situation is described in Table 3.

Table 3

Number of Nontract Residents Required to Fill the Third and Fourth Shelters in Tract #12

Facility No.	Shelter Capacity After 3,324 Tracks #12 Residents Take Shelter	Number of Nontract Residents who Must Take Shelter Before One Additional Shelter is Filled	Number of Remaining Shelters Having Excess Capacity
1	0	0	Two
2	0	0	Two
3	1,487	2,974	One
4	1,905	418	None

6. It is assumed that a person seeking shelter will visit any particular shelter only once; i.e., having once visited a full shelter, he will not return to it. The consequence of this assumption is that the universe of shelters a person might visit grows smaller each time he finds a full shelter. For example, suppose a person seeks shelter in a tract having 8 shelters under the conditions where there are 4 full shelters and 4 shelters with excess capacity. He has a 50% chance of finding a shelter with excess capacity upon arrival at the first shelter. If this first shelter happened to be full, he then has 3 full shelters and 4 shelters with excess capacity to choose from. Therefore, his chances of finding space in a shelter on his second try are 4 out of 7 or 57%.

7. During each period where the number of shelters having excess capacity remains constant, it is assumed that the mean distance a new arrival must travel from the tract border to a shelter having excess capacity also remains constant. This mean distance is represented by equation (3):

$$D = R + \frac{n}{N}(d) + \left(\frac{n-1}{N-1}\right)\left(\frac{n}{N}\right)(d) + \left(\frac{n-2}{N-2}\right)\left(\frac{n-1}{N-1}\right)\left(\frac{n}{N}\right)(d) \ldots \quad (3)$$

where: n = number of shelters with no remaining capacity.

N = number of shelters having protection factors of 100 or better.

R = mean distance from the tract border to the first shelter in miles (equation (2), page 6-3).

d = distance between adjacent shelters in miles (equation (1), page 6-3).

D = mean distance from tract border to a shelter having excess capacity, in miles.

In the example used here it is necessary to calculate two values of D. The first applies to that group of people seeking shelter under the initial conditions where two of the four possible shelters have excess capacity. In this case:

n = two shelters have no remaining capacity.

N = four shelters having possible protection factors of 100 or better.

R = .20 miles.

d = .52 miles.

and, therefore,

D_1 = .55 miles.

Similarly, when the third shelter fills up, n now equals three, and D_2 is calculated as 1.00 miles.

8. **Each time a shelter is filled, it is assumed that no additional persons enter shelter for a period of time represented by equation (4):**

$$t = \frac{D_x - D_y}{r} \qquad \textbf{(4)}$$

where: t = time in minutes during which no one enters shelter.

D_x = mean distance in miles from tract border to an empty shelter immediately following the filling up of the last shelter (equation (3), page 6-5).

D_y = mean distance in miles from tract border to an empty shelter immediately previous to the last shelter filling up (equation (3), page 6-5).

r = rate at which people walk to shelter within the tract (required data page 6-3).

In the case of tract #12, this situation occurs only once, at the point in time when the third shelter fills up. Under these conditions, $D_y = D_1 = .55$ miles. $D_x = D_2 = 1.00$ miles, and $r = .05$ miles per minute. Consequently, $t = 9.00$ minutes.

III. APPLICATION OF THE MODEL

Using the required data and assumptions presented in Sections I and II, the rate at which shelter spaces are filled by persons arriving from other tracts can be calculated. The step by step process by which these calculations are made for tract #12, City of Stamford, Connecticut, is presented in Table 4. Combining the information in columns "C" (time) and "E" (total people in shelter) of Table 4, it is possible to create a graphic representation of nontract residents sheltered as a function of time. Such a graph for tract #12 is presented on page 6-9. Similar graphs are constructed for each census tract having a shelter capacity greater than the resident population. For the City of Stamford, Connecticut, this is the case for tracts 1, 11, 12, 16, 17, 18, 22, and 23. By combining graphs of this type with similar graphs showing entrance into shelter as a function of time for tract residents, it is possible to develop a composite graph for an entire area or city. Such a graph for the City of Stamford, Connecticut. is presented on page 20 in Volume I.

Table 4

Step by step process by which shelters in Tract #12, City of Stamford, Conn., not filled by Tract #12 residents are filled by people arriving from other tracts.

(A) Event No.	(B) Event	(C) Time (minutes)	(D) People entering shelters during time period	(E) Total No. people in shelter at end of time period
1	People begin entering Tract #12 at rate of 59.43/min. (Table 2, page 6-2).	0	0	0
2	First person entering Tract #12 enters shelter at: $\frac{D_1}{r} = \frac{.55 \text{ miles}}{.05 \text{ miles/min.}} = 11.0$ minutes where: D_1 = Mean distance a person travels from time he enters Tract #12 until he enters a shelter having excess capacity [Equation (3), page 6-5]. r = Assumed rate at which people walk to shelters (page 6-3).	11.0	0	0
3	2974 people must enter shelters in Tract #12 before Shelter #3 is filled up (Table 3, page 6-4). The first 1355 people of this group enter at rate of 59.43/min. (Table 2, page 6-2). Therefore it takes 22.8 min. for these first 1355 people to enter shelters (Table 2, page 6-2).	11.0-33.8	1355	1355
4	No one enters tract until 41.4 minutes (Table 2, page 6-2).	41.4	0	1355
5	At 41.4 min. people again begin entering Tract #12 but at new rate of 121.63/min. (Table 2, page 6-2).	41.4	0	1355
6	People again begin entering shelter at time 41.4 + 11.0 = 52.4 min. $\frac{D_1}{r} = \frac{.55 \text{ miles}}{.05 \text{ miles/min.}} = 11.0$ minutes where: D_1 and r are as described for Event #2.	52.4	0	1355

Table 4 (Continued)

(A) Event No.	(B) Event	(C) Time (minutes)	(D) People entering shelters during time period	(E) Total No. people in shelter at end of time period
7	From Event #3, 1355 out of the 2974 people required to fill the third shelter have already arrived. Therefore 1619 more people can enter shelters before the third shelter is filled. These 1619 people enter shelters at rate of 121.63/min (Table 2, page 6-2). Therefore it takes 13.3 minutes for these people to enter shelters.	52.4-65.7	1619	2974
8	The third shelter (out of the 4 in the tract) is now filled up.	65.7	0	2974
9	The mean distance from the Tract #12 border to a shelter with excess capacity has increased from .55 miles to 1.00 miles (Equation 3, page 6-5). This represents an increased distance of .45 miles. With a uniform walking rate of .05 mi./min. (Assumption 3, page 6-3) it takes an added 9.0 minutes for people to arrive at a shelter having excess capacity. Therefore according to Assumption 8, page 6-6, no one enters a shelter for the next 9.0 minutes.	65.7-74.7	0	2974
10	The first person in the last group of 418 to be sheltered in Tract #12 is ready to enter Shelter #4.	74.7	0	2974
11	The final 418 people now begin to fill up Shelter #4 at the rate of 121.63 people/min. (Table #2, page 6-2). It therefore takes 3.4 minutes from the time the first person in this group enters Shelter #4 to the time the last person enters a shelter.	74.7-78.1	418	3392

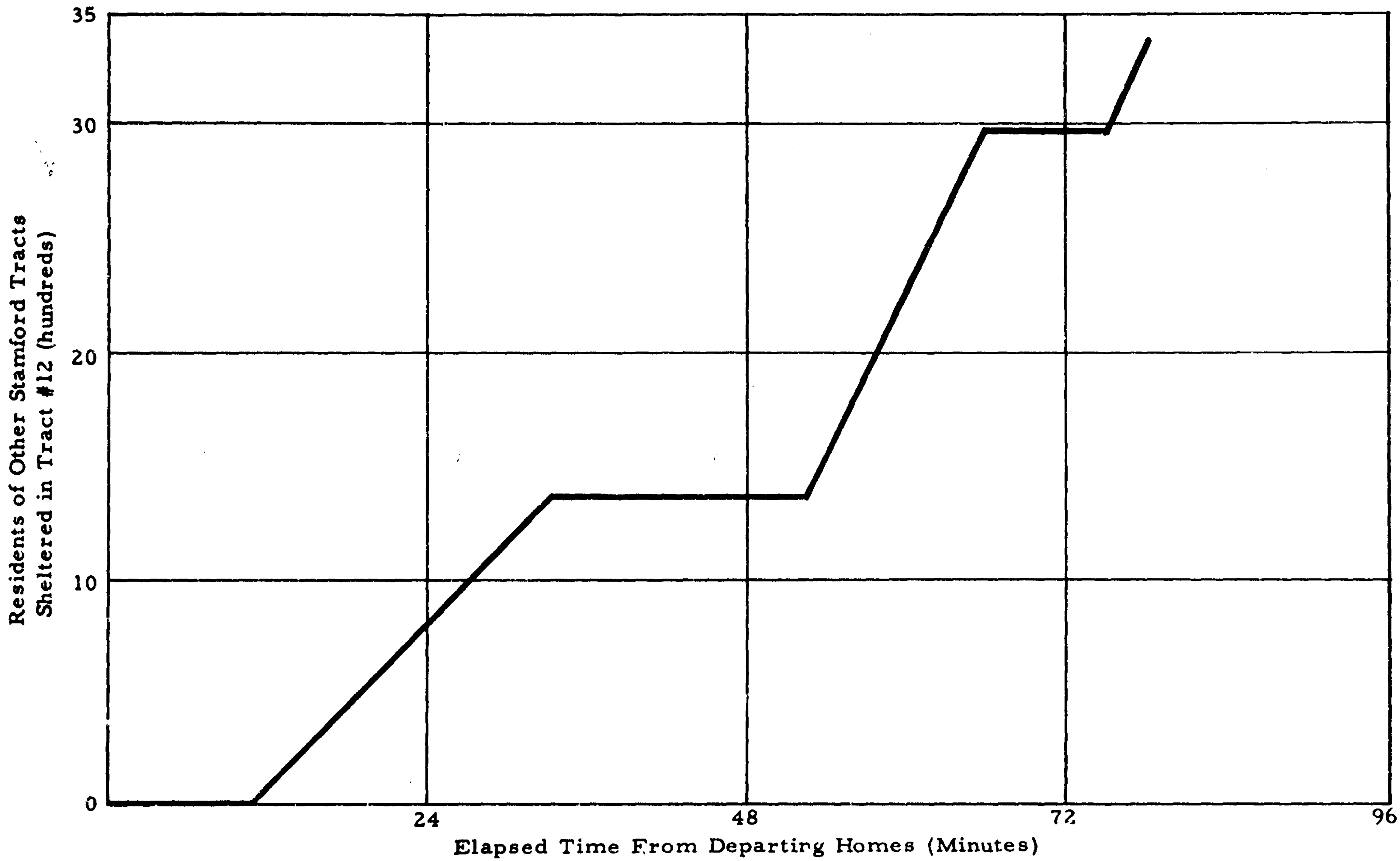

Figure 1. Residents of Other Stamford Tracts Sheltered in Tract #12 versus Elapsed Time from Departing Homes, Assuming 40 mph Driving Speed, 3 mph Walking Speed, One Lane Traffic, and No Individual Shelter Assignment.

APPENDIX 7

Mathematical Description of Fire Probabilities

1. The State Equations

It is postulated that initially the depopulated region contains N "sources" of potential fires: typical examples are unextinguished cigarettes, operating electrical appliances, automatic heating systems, and structures inadequately protected from lightning. During any time interval $[0, t]$ one of three events associated with such a source may occur.

(a) The source may lose its potential without initiating a self-sustaining fire: the cigarette may burn out harmlessly, the fuse in the circuit feeding the appliance may blow, etc.

(b) The potential may be converted into actuality: the source may initiate a self-sustaining fire in its surroundings.

(c) The potential may continue unmodified: the heating system may operate and remain a potential hazard, the danger from electrical storms continues unmodified, etc.

These events occur at random points in time. As stated in the main text, the assumption used in this development is that the probability that such an event occurs in an interval $[t, t+h]$, given that nothing which makes the event impossible, occurred during $[0, t]$, is proportional to the duration of the interval, h, with some undefined dependence upon terms of order h^2 or larger. The formal application of this assumption to the three cases listed is as follows.

(a) Given that neither an event of type (a) nor an event of type (b) occurred during $[0, t]$, the probability that the event of type (a) occurs in $[t, t+h]$ is $ah + o(h)$. Here, "a" is a proportionality constant, and $o(h)$ represents terms of order h^2 or higher.

(b) Given that neither an event of type (a) nor an event of type (b) occurred in $[0, t]$, the probability that an event of type (b) will occur in $[t, t+h]$ is $bh + o(h)$, where "b" is a constant of proportionality.

(c) It follows that under the same given conditions the probability that neither type of event occurs in $[t, t+h]$ is $1 - (a+b)h + o(h)$.

It is convenient to describe any possible combination of events in the region by defining "states of the region" as follows.

state 0:	no events of either kind for any of the N sources
state 1:	one event of type (a), none of type (b)
.	
.	
state n:	n events of type (a), none of type (b)
.	
.	
state N:	N events of type (a), none of type (b)
state Z:	one event of type (b), number of prior events of type (a) not specified.

The region has two terminal states. If it is in state N no further events can occur, and if it is in state Z (on fire) additional events of type (a) are no longer significant.

The procedure used by Feller* is as follows. Let $P_n(t)$ represent the probability that the region is in state n, $n \neq z$, at time t. The probability that the region is in state n at $t+h$ is the probability of the following compound event:

(a) n events of type (a) occurred during $[0, t]$ and none during $[t, t+h]$: the probability of this event is $P_n(t)\,[1 - (a+b)h + o(h)]^{N-n}$ since there are $(N-n)$ sources available;

(b) $(n-1)$ events of type (a) occurred in $[0, t]$, one event of type (a) occurred in $[t, t+h]$ and none of type (b) in $[t, t+h]$: the probability of this event is $P_{n-1}(t)[ah + o(h)]$.

Accordingly the probability of the compound event is

$$(1) \qquad P_n(t+h) = P_n(t)[1 - (N-n)ch + o(h)] + P_{n-1}(t)[ah + o(h)]$$

where $c = a + b$ has been introduced for convenience.

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*Feller, William, An Introduction to Probability Theory and its Applications, Vol. I, John Wiley and Sons, New York; 1950. In particular, see Chapter 17.

Rearranging terms

$$(2) \qquad \frac{P_n(t+h) - P_n(t)}{h} = -(N-n)cP_n(t) + aP_{n-1}(t) + \frac{o(h)}{h}$$

In the limit as h approaches zero, the term on the left becomes the derivative of P_n and the last term on the right approaches zero. Thus

$$(3) \qquad P_n'(t) + (N-n)cP_n(t) = aP_{n-1}(t) \quad ; \qquad n = 1, 2, \ldots, (N-1)$$

where the prime indicates differentiation with respect to t. Differential equations for $P_0(t)$ and $P_N(t)$ can be derived by an analogous process modified to take account of the fact that there are no transitions to state zero and no transitions from state N. The resultant equations are

$$(4) \qquad P_0'(t) + NcP_0(t) = 0$$

and

$$(5) \qquad P_N'(t) = aP_{N-1}(t)$$

(A differential equation for $P_Z(t)$ can also be derived in this way, but a simpler procedure is to compute this from $\sum_{n=0}^{Z} P_n(t) = 1$.) The initial conditions to be satisfied by these equations are derived from the assumption that no events occur before t = 0: these are $P_o(0) = 1$, and $P_n(0) = 0$. It is shown in Supplement A to this Section that the solutions to these equations are

$$(6) \qquad P_n(t) = \frac{r^n}{n!} \exp[-(N-n)ct]$$

where

$$(7) \qquad r = \frac{a}{c}[1 - \exp(-ct)]$$

For later reference it is noted that

$$(8) \qquad r' = a \exp(-ct)$$

2. The Physical Significance of the Proportionality Constants

In the preceding discussion, the constants a, b, and $c = a + b$, were introduced as constants of proportionality. Considered dimensionally, these constants represent rates; that is a number per unit of time. Referring back to the definition of "a," it can be seen that if this is increased, the probability of a non-fire producing event during a given interval is increased. It follows that "a" represents the rate at which such events occur. Similarly, "b" represents the rate at which fire-producing events occur. A more precise characterization of the significance of these constants can be obtained as follows.

Consider a region containing only one source; i.e., $N = 1$. Let the time scale be divided into intervals of length h so that $t_i = ih$. The life (or duration) of state 0 can be assigned the value t_i if no events occur during $[0, t_i]$ and an event of type a or type b occurs during $[t_i, t_i + h]$. Since the probability of this event is ch the expected value of the state 0 lifetime is

$$(9) \qquad T_o = \sum_{0}^{\infty} t_i P_o(t_i) ch$$

In the limit, as h approaches zero,

$$(10) \qquad T_o = c\int_0^{\infty} tP_o(t)\,dt = c\int_0^{\infty} t\exp(-ct)\,dt = (1/c)$$

If an ensemble of regions with $N = 1$ is considered, T_o can also be called the mean accident-free time and the ergodic implication is that $(1/c)$ can also be called the mean time between accidents if the state is restored to zero after each accident. It is also consistent to interpret c as the average accident rate.

The same rationale leads to the conclusion that the expected value of the time required to reach the state P_1, after which no accident is possible, is given by

$$(11) \qquad T_1 = a\int_0^{\infty} tP_o(t)\,dt = (a/c^2)$$

and the expected value of the time required for a fire to start (i.e., to reach state Z) is given by

(12) $$T_Z = b\int_0^{\infty} tP_o(t)\,dt = (b/c^2)$$

In other words, (c^2/b) is the average rate at which the accident potential is dissipated harmlessly, and (c^2/a) is the average rate at which fires start.

The rationale can be extended to the general case. State Z can be reached by a transition from any state except $n = N$. Since the probability of a transition from any state n/N to state Z is $(N-n)bh$, the expected value of the time before fire occurs is

(13) $$T_Z = b \sum_{n=0}^{N-1} (N-n) \int_0^{\infty} tP_n(t)\,dt$$

where the order of integration and summation have been interchanged. This quantity is evaluated in Supplement B, giving

(14) $$T_Z = b/Nc^2$$

Alternately, Nc^2/b is the average rate at which fires start in an ensemble of regions with N sources in each region.

3. The Cluster

The preceding discussion has described events in a region containing N potential sources of spontaneous ignition, and which burns completely if one of these sources starts a self-sustaining fire. It is convenient to refer to such a region as a cluster: this name suggests that the structures and ignition points are distributed in a way which makes fire spread through the region inevitable if no control is attempted. A more complete physical characteristic of a cluster must await further developments by the experts studying fire spread mechanisms. It is clear that a cluster may be an entire downtown region or a single house in the suburbs. The utility of this concept will become apparent later in the discussion when it is pointed out that fire spread can be characterized by considering the probability that a burning cluster ignites a contiguous one. The statistical description of such events can be developed despite the lack of a clear physical definition.

4. The State Equations for a Cluster

The condition of a cluster can be characterized by defining three states as follows:

State 0 - no fire has occurred
State 1 - cluster burning
State 2 - fire burned out

It is recognized that the words "burning" and "out" used in defining States 1 and 2 are not precise. In a more exact analysis than is of interest here, it would certainly be appropriate to introduce the states "violent burning" and "residual burning" discussed by Chandler, et al.* The discussion is materially simplified by ignorning this distinction for the moment.

Using this definition of the states, the events of interest can be characterized as transitions from one state to another. The transition $0 \longrightarrow 1$ represents the event: the cluster catches fire. The transition $1 \longrightarrow 2$ represents the burning out of the fire. Clearly, there are no transitions from State 2.

Using these assumptions and definitions and the analytical procedure described in Section 4, the following results are obtained.

$$P_0(t) = e^{-at} \tag{15}$$

describes the probability that the cluster has not caught fire by time t given that it was not on fire at time 0. The probability that the cluster is burning at any time t is given by

$$P_1(t) = \frac{a}{b-a}\left(e^{-at} - e^{-bt}\right) \tag{16}$$

The analogous procedure for $P_2(t)$ gives

$$P_2(t) = 1 - \frac{1}{b-a}\left(be^{-at} - ae^{-bt}\right) \tag{17}$$

As a check it is noted that

$$P_0(t) + P_1(t) + P_2(t) = 1 \tag{18}$$

*Chandler, C. C., Storey, T. G., and Tangren, C. D. Prediction of Fire Spread Following Nuclear Explosions. U. S. Forest Service Research Paper PSW-5, 1963.

5. Physical Interpretation

Using the established definitions, the probability that the transition from 0 to 1 occurs in the interval $[t, t+h]$ is $ahP_0(t)$. The expected value of the time at which this transition takes place is

$$E(T_0) = a \sum_{i=0}^{n} tP_0(t)h_i \tag{19}$$

Converting this to an integral by letting n approach infinity and the largest h_i approach zero.

$$E(T_0) = a \int_0^{\infty} P_0(t)\,dt = (1/a) \tag{20}$$

Thus a is the reciprocal of the time before fire starts or of the time the cluster is in State 0. If an ensemble of independent clusters is considered, this can also be interpreted as the mean (or average) lifetime of the fire-free State 0. Alternately, "a" itself can be interpreted as the average rate at which fires start in the ensemble of clusters. It is clear that any action taken before depopulation (e.g., inspection to reduce the number of potential ignition points in the clusters) will decrease a or increase (1/a).

An analogous analysis yields that the expected value of the time at which the transition from State 1 to State 2 occurs is given by

$$E(T_1) = b \int_0^{\infty} tP_1(t)\,dt = \frac{a+b}{ab} \tag{21}$$

The difference between these two quantities is the expected value of the time between the two transitions, or the expected value of the lifetime of the burning State 1. This is

$$\frac{a+b}{ab} - \frac{1}{a} = \frac{1}{b} \tag{22}$$

Thus b is the reciprocal of the average duration of the fire. It can also be interpreted as a burn-out rate but this does not appear to be very helpful. There is little or nothing that can be done before depopulation which may affect b materially, b would appear to be a function of fuel density and the

size of the cluster with some possible statistical variations induced by rain or its absence, wind conditions, etc. Because of this, a convenient way to normalize the description is to let $(1/b)$ equal one arbitrary unit of time. Some typical results when $b = 1$ are shown in Figure 2 of the text.

6. A Rough Estimate of the Hazard

Finally, it is interesting to speculate about the possible values of a and b. If it is recalled that a can be interpreted as the mean rate at which fires occur, an estimate of its value for normally occupied areas can be obtained from statistics on the number of fires which actually occur. As a typical example, the number of actual fires in Los Angeles in the year starting 1 July 1958 is given* as 13, 933. For this case, $a = 1.6$ per hour, or equivalently $(1/a)$ the mean time between fires is 0.625 hours. We have not found any useful guide for extrapolation from this number to a figure which might apply during depopulation. Even an optimistic doubling of this mean time however leads to the conclusion that this is a serious hazard which may render any other measures for protecting the population relatively unimportant, especially if the fallout shelters themselves are threatened by the spreading fires.

7. An Initial Discussion of Fire Spread

The analysis developed in Section 4 can be extended to include consideration of fire spread by introducing a modified definition of state, and using the concept of a cluster. The essential characteristic of a cluster is that there is a finite probability that it will burn out without spreading to neighboring areas. Conceptually, any region can be subdivided into M such clusters. The state of the region can then be represented as a M-tuple $(i_1, i_2, \ldots, i_M)$, in which each element represents the state of the corresponding cluster. If the cluster states defined in Section 5 are used, $i = 0$ indicates no ignition, $i = 1$ indicates active burning, and $i = 2$ indicates burned out, and there are then 3^M possible states. (More detailed definitions of cluster states based on the N sources in each cluster can also be used.) Both initial ignition and spread between clusters can then be described analytically as transitions from one of these states to another.

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*A Study of Fire Problems, Publication 949, National Academy of Sciences, National Research Council, Washington, D. C., 1961. (This is a report of a study held at Woods Hole, Massachusetts, under the auspices of the Committee on Fire Research of the National Academy of Sciences.)

As a comprehensible example which can easily be generalized, suppose that $M = 2$; i.e., that the region is divided into two clusters. Then the state (0, 0) designates that no fires have started in either cluster. The transition (0, 0) to (0, 1) indicates that a fire has started in the second cluster which is now burning. The probability of such an initial transition is that discussed for transitions from 0 to 1 in a single cluster; i.e., $a_2 h + o(h)$ where the subscript identifies the cluster. A subsequent transition may be from (0, 1) to (1, 1). This can occur in one of two ways: there may be spontaneous ignition in cluster 1, or the fire may spread from cluster 2 to cluster 1. This second possibility can be described by introducing a probability of the form $g_{ij} h + o(h)$, where g is completely analogous to the parameters a and b used in the preceding discussion. The probability of the transition from (0, 1) to (1, 1) in $[t, t+h]$ is then $(a_1 + g_{21})h + o(h)$. Other possible transitions can be analyzed in the same way. When higher order terms are ignored, the possible transitions are those indicated by the arrows in Figure 2.

When the spread of fire is considered in this way, it is clear that the methods described in Section 4 are directly applicable to the derivation of simultaneous linear differential equations describing the probability $P_{ij}(t)$ that the region is in any of the nine possible states at any time t. From this description, it is also clear that if the description is quantized, as would normally be done if a digital computer is to be used for the calculations, the process is a simple Markov process; namely, a random walk with one absorbing state, (2, 2). The matrix of transition probabilities for this particular example is of the form shown in Figure 3.

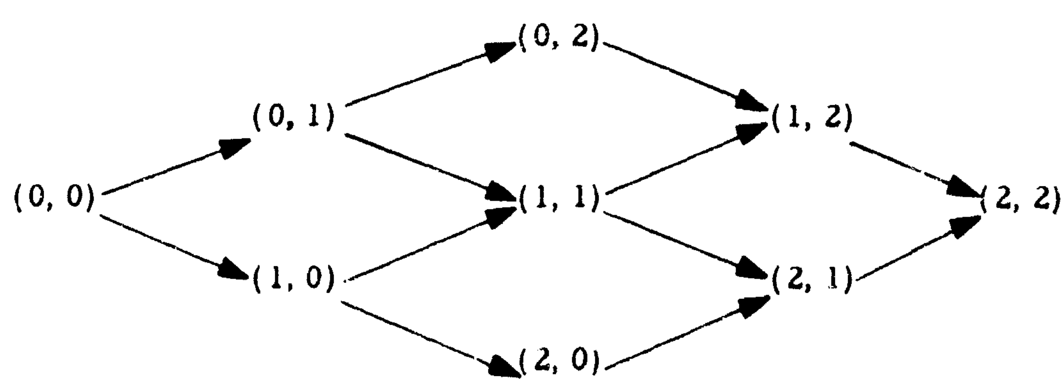

Figure 2. Possible transitions for region with two clusters.

	0, 0	0, 1	1, 0	1, 1	0, 2	2, 0	1, 2	2, 1	2, 2
0, 0	$1-a_1-a_2$	a_2	a_1	0	0	0	0	0	0
0, 1	0	$1-a_1-b_2$	0	a_1	b_2	0	0	0	0
1, 0	0	0	$1-a_2-b_1$	a_2	0	b_1	0	0	0
1, 1	0	0	0	$1-b_1-b_2$	0	0	b_2	b_1	0
0, 2	0	0	0	0	$1-a_1$	0	a_1	0	0
2, 0	0	0	0	0	0	$1-a_2$	0	a_2	0
2, 2	0	0	0	0	0	0	0	0	1

Figure 3. Matrix of transition probabilities for region with two clusters.

SUPPLEMENT A TO APPENDIX 7

Solution to State Equations

To demonstrate that equation (16) provides solutions to equations (3), (4), and (5), it is sufficient to substitute the indicated quantities into the equations and check that the result is an identity. The asserted solutions are

$$(6) \qquad P_n = \frac{r^n}{n!} e^{-(N-n)ct}$$

where

$$(7) \qquad r = \frac{a}{c}(1 - e^{-ct})$$

and

$$(8) \qquad r' = \frac{dr}{dt} = ae^{-ct}$$

For n = o, equation (6) gives

$$(A\text{-}1) \qquad P_o = e^{-Nct}$$

differentiating

$$(A\text{-}2) \qquad P'_o = Nce^{-Nct}$$

these clearly satisfy equation (4).

For n = N, equation (6) gives

$$(A\text{-}3) \qquad P_N = \frac{r^N}{N!}$$

differentiating

$$(A\text{-}4) \qquad P'_N = \frac{r^{N-1}}{(N-1)!} ae^{-ct}$$

For $n = N-1$, equation (6) gives

$$aP_{N-1} = \frac{ar^{N-1}}{(N-1)!} e^{-ct} \tag{A-5}$$

Since these two are equal, equation (5) is satisfied.

To check that equation (3) is satisfied, equation (6) is differentiated, giving

$$P'_n = \frac{1}{n!} r^{n-1} e^{-(N-n)ct} \left[nae^{-ct} - (N-n)cr\right] \tag{A-6}$$

Also, from equation (6)

$$(N-n)\,cP_n = \frac{1}{n!} r^{n-1} e^{-(N-n)ct} \left[(N-n)\,cr\right] \tag{A-7}$$

Adding these as indicated by equation (3) yields the quantity

$$\frac{1}{(n-1)!} ar^{n-1} e^{-(N-n+1)ct} \tag{A-8}$$

Since this is a P_{n-1} from equation (6), equation (3) is satisfied for

$n = 1, 2, \ldots, N-1$

Finally, it can be checked by inspection that the initial conditions are satisfied.

SUPPLEMENT B TO APPENDIX 7

Mean Time to Ignition in Regions with N Sources

T_z is given by equation (13); i.e.,

$$(13) \qquad T_z = b \sum_{n=0}^{N-1} (N-n) \int_0^\infty t\,P_n(t)\,dt$$

It will be shown that it is only necessary to develop the first few terms of this summation. Define

$$(B\text{-}1) \qquad 1(n;N) = (N-n)\,c \int_0^\infty t\,P_a\,dt$$

In particular, for n = 0

$$(B\text{-}2) \qquad 1(0;N) = Nc \int_0^\infty t\,P_o\,dt$$

Using equation (3) for P_o

$$(B\text{-}3) \qquad 1(0:N) = -\int t\,P'_o\,dt$$

Integrating by parts

$$(B\text{-}4) \qquad 1(0;N) = [-t\,P_o]_0^\infty + \int_0^\infty P_o\,dt$$

The first term vanishes since $P_o(\infty) = 0$. Expressing P_o explicitly by using equation (B-5) for n = 0.

$$(B\text{-}5) \qquad 1(0;N) = \int_0^\infty e^{-Nct}\,dt = \frac{1}{Nc}$$

The process is repeated for n = 1.

(B-6) $$I(1;N) = (N-1)c\int_0^\infty t\,P_1\,dt$$

Using equation (4) for n = 1

(B-7) $$I(1;n) = a\int_0^\infty tP_o\,dt - \int_0^\infty tP'_1\,dt$$

The first term is $(a/Nc)I(o;N)$ and is evaluated from (B-5). The second term is integrated by parts using that as $t = 0$, $P_1(t) = 0$, exponentially dominating t. This yields

(B-8) $$I(1:N) = \frac{a}{N^2c^2} + \int_0^\infty P_1\,dt$$

Using equation (4) with n = 1

(B-9) $$(N-1)c\int_0^\infty P_1\,dt = a\int_0^\infty P_o\,dt - \int_0^\infty P'_1\,dt$$

The second term vanishes since $P_1(0) = P_1(\infty) = 0$. The first term was evaluated in obtaining (B-5). Thus

(B-10) $$\int_0^\infty P_1\,dt = \frac{a}{N(N-1)c^2}$$

Substituting this into (B-8)

(B-11) $$I(1;N) = \frac{(2N-1)a}{N^2(N-1)c^2}$$

Repeating the process for n = 2

(B-12) $$I(2:N) = \frac{(3N^2-6N+2)a^2}{N^2(N-1)^2(N-2)c^3}$$

From (13) and (B-1)

(B-13) $$T_z = \frac{b}{c}\,[I(o;N) + I(1;N) + I(2;N) + \ldots]$$

Using the values obtained in (B-5), (B-11), and (B-12)

$$\text{(B-14)} \qquad T_z = \frac{b}{Nc^2} \quad 1 + \frac{(2N-1)a}{N(N-1)c} + \frac{(3N^2 - 6N + 2)}{N(N-1)^2(N-2)} + \ldots$$

Since a < c, and the denominators are increasingly higher order than the numerators, the higher order terms can be neglected.

A first approximation, therefore, is

$$\text{(14)} \qquad T_z = \frac{b}{Nc^2}$$